Cuaderno de notas de trabajo

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Cuaderno 51

Principios Variacionales.

Variación de la ds de la Relatividad Especial.

Hipótesis:

1) las variaciones libres son las de las coordenadas.

2) la variación de ds es función de las variaciones de las coordenadas.

$$\boxed{ds^2 = \Delta_{ij}\, dx^i dx^j}$$

$$2\, ds\, \delta(ds) = 2\, \Delta_{ij}\, dx^i\, d(\delta x^j)$$

$$\delta(ds) = \Delta_{ij} \frac{dx^i}{ds}\, d(\delta x^j)$$

$$\boxed{\frac{dx^i}{ds} = v^i}$$

$$\boxed{\delta(ds) = \Delta_{ij}\, v^i\, d(\delta x^j)}$$

$$\boxed{\boxed{\delta(ds) = \hat{v} \cdot d(\delta \hat{r})}}$$

¡$\delta \hat{r}$ es una variación libre!

Variación del cuadrivector velocidad.

$$\delta v^i = \delta \frac{dx^i}{ds} = \frac{d(\delta x^i)}{ds} - dx^i \frac{\hat{v} \cdot d(\delta \hat{r})}{ds^2}$$

$$\delta v^i = \frac{d}{ds}(\delta x^i) - v^i \left(\hat{v} \cdot \frac{d}{ds} \delta \hat{r} \right)$$

$$\boxed{\delta \hat{v} = \frac{d}{ds}(\delta \hat{r}) - \left(\hat{v} \cdot \frac{d}{ds}(\delta \hat{r}) \right) \hat{v}}$$

$$\boxed{\delta v^i = \frac{d}{ds}(\delta x^i) - \left[\Delta_{jk} v^j \frac{d}{ds}(\delta x^k) \right] v^i}$$

$$L\,ds = A + A_{i_1}\,dx^{i_1} + A_{i_1 i_2} v^{i_1} dx^{i_2} + A_{i_1 i_2 i_3} v^{i_1} v^{i_2} dx^{i_3} + A_{i_1 i_2 i_3 i_4} v^{i_1} v^{i_2} v^{i_3} dx^{i_4} + \ldots$$

$$\boxed{L\,ds = \sum_i A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} dx^{i_n}}$$

$$\boxed{L\,ds = \sum_i A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n}\, ds}$$

$$\boxed{A_{i_1 i_2 i_3 \cdots i_\alpha \cdots i_\beta \cdots i_{n-1} i_n} = A_{i_1 i_2 i_3 \cdots i_\beta \cdots i_\alpha \cdots i_{n-1} i_n}}$$

$$\delta\, A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3}\, ds = \left\{ \frac{\partial A_{i_1 i_2 i_3}}{\partial x^j} v^{i_1} v^{i_2} v^{i_3}\, \delta x^j\, ds + 3 A_{i_1 i_2 i_3} v^{i_1} v^{i_2}\, d(\delta x^{i_3}) \right.$$

$$- 3 A_{i_1 i_2 i_3} \Delta_{i_4 i_5} v^{i_1} v^{i_2} v^{i_3} v^{i_4}\, d(\delta x^{i_5})$$

$$\left. + A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3} \Delta_{i_4 i_5} v^{i_4}\, d(\delta x^{i_5}) \right\}$$

$$\boxed{\delta A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3}\, ds = \left[\frac{\partial A_{i_1 i_2 i_3}}{\partial x^{i_4}} v^{i_1} v^{i_2} v^{i_3}\, \delta x^{i_4}\, ds + 3 A_{i_1 i_2 i_3} v^{i_1} v^{i_2}\, d(\delta x^{i_3}) - 2 A_{i_1 i_2 i_3} \Delta_{i_4 i_5} v^{i} v^{i_2} v^{i_3} v^{i_4}\, d(\delta x^{i_5}) \right]}$$

$$\delta A_{i_1 i_2 i_3 \cdots i_n} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_n}\, ds = \left[\frac{\partial A_{i_1 i_2 \cdots i_n}}{\partial x^{i_{n+1}}} v^{i_1} v^{i_2} \cdots v^{i_n}\, \delta x^{i_{n+1}}\, ds + n A_{i_1 i_2 \cdots i_n} v^{i_1} v^{i_2} \cdots v^{i_{n-1}}\, d(\delta x^{i_n}) - (n-1) A_{i_1 i_2 \cdots i_n} \Delta_{i_{n+1} i_{n+2}} v^{i_1} v^{i_2} \cdots v^{i_n} v^{i_{n+1}}\, d\delta x^{i_3} \right]$$

$(d\delta x^{i_{n+2}})$

Variación del elemento de arco ds.

$$ds^2 = \Delta_{ij}\, dx^i dx^j$$

$$2\, ds\, \delta(ds) = 2\Delta_{ij}\, dx^i\, d(\delta x^j)$$

$$\delta(ds) = \Delta_{ij} \frac{dx^i}{ds}\, d(\delta x^j)$$

$$\boxed{\delta(ds) = \Delta_{ij}\, v^i\, d(\delta x^j)}$$

$$\boxed{\delta(ds) = \hat{v} \cdot d(\delta \hat{r})}$$

Variación del cuadrivector velocidad $\hat{v}$

$$v^i = \frac{dx^i}{ds}$$

$$\delta v^i = \frac{d}{ds}(\delta x^i) - \frac{dx^i}{ds^2}\left[\hat{v} \cdot d(\delta \hat{r})\right]$$

$$\boxed{\delta v^i = \frac{d}{ds}(\delta x^i) - v^i \left[\Delta_{jk}\, v^j \frac{d}{ds}(\delta x^k)\right]}$$

$$\boxed{\delta \hat{v} = \frac{d}{ds}(\delta \hat{r}) - \hat{v}\left[\hat{v} \cdot \frac{d}{ds}(\delta \hat{r})\right]}$$

$$A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3}$$

$$A_{111} v^1 v^1 v^1 + A_{112} v^1 v^1 v^2 + A_{113} v^1 v^1 v^3 + A_{114} v^1 v^1 v^4$$
$$+ A_{121} v^1 v^2 v^1 + A_{122} v^1 v^2 v^2 + A_{123} v^1 v^2 v^3 + A_{124} v^1 v^2 v^4$$
$$+ A_{131} v^1 v^3 v^1 + A_{132} v^1 v^3 v^2 + A_{133} v^1 v^3 v^3 + A_{134} v^1 v^3 v^4$$
$$+ A_{141} v^1 v^4 v^1 + A_{142} v^1 v^4 v^2 + A_{143} v^1 v^4 v^3 + A_{144} v^1 v^4 v^4$$
$$+ A_{211} v^2 v^1 v^1 + A_{212} v^2 v^1 v^2 + A_{213} v^2 v^1 v^3 + A_{214} v^2 v^1 v^4$$
$$+ A_{221} v^2 v^2 v^1 + A_{222} v^2 v^2 v^2 + A_{223} v^2 v^2 v^3 + A_{224} v^2 v^2 v^4$$
$$+ A_{231} v^2 v^3 v^1 + A_{232} v^2 v^3 v^2 + A_{233} v^2 v^3 v^3 + A_{234} v^2 v^3 v^4$$
$$+ A_{241} v^2 v^4 v^1 + A_{242} v^2 v^4 v^2 + A_{243} v^2 v^4 v^3 + A_{244} v^2 v^4 v^4$$
$$+ A_{311} v^3 v^1 v^1 + A_{312} v^3 v^1 v^2 + A_{313} v^3 v^1 v^3 + A_{314} v^3 v^1 v^4$$
$$+ A_{321} v^3 v^2 v^1 + A_{322} v^3 v^2 v^2 + A_{323} v^3 v^2 v^3 + A_{324} v^3 v^2 v^4$$
$$+ A_{331} v^3 v^3 v^1 + A_{332} v^3 v^3 v^2 + A_{333} v^3 v^3 v^3 + A_{334} v^3 v^3 v^4$$
$$+ A_{341} v^3 v^4 v^1 + A_{342} v^3 v^4 v^2 + A_{343} v^3 v^4 v^3 + A_{344} v^3 v^4 v^4$$
$$+ A_{411} v^4 v^1 v^1 + A_{412} v^4 v^1 v^2 + A_{413} v^4 v^1 v^3 + A_{414} v^4 v^1 v^4$$
$$+ A_{421} v^4 v^2 v^1 + A_{422} v^4 v^2 v^2 + A_{423} v^4 v^2 v^3 + A_{424} v^4 v^2 v^4$$
$$+ A_{431} v^4 v^3 v^1 + A_{432} v^4 v^3 v^2 + A_{433} v^4 v^3 v^3 + A_{434} v^4 v^3 v^4$$
$$+ A_{441} v^4 v^4 v^1 + A_{442} v^4 v^4 v^2 + A_{443} v^4 v^4 v^3 + A_{444} v^4 v^4 v^4$$

$$A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3}$$

$$A_{111}(v^1)^3 + A_{222}(v^2)^3 + A_{333}(v^3)^3 + A_{444}(v^4)^3$$
$$+3A_{112}(v^1)^2 v^2 + 3A_{113}(v^1)^2 v^3 + 3A_{114}(v^1)^2 v^4$$
$$+3A_{122} v^1 (v^2)^2 + 3A_{223}(v^2)^2 v^3 + 3A_{224}(v^2)^2 v^4$$
$$+3A_{133} v^1 (v^3)^2 + 3A_{233} v^2 (v^3)^2 + 3A_{334}(v^3)^2 v^4$$
$$+3A_{144} v^1 (v^4)^2 + 3A_{244} v^2 (v^4)^2 + 3A_{344} v^3 (v^4)^2$$
$$+6A_{123} v^1 v^2 v^3 + 6A_{124} v^1 v^2 v^4 + 6A_{134} v^1 v^3 v^4$$
$$+6A_{234} v^2 v^3 v^4$$

$$\frac{\partial}{\partial v^3} A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3} = \left[\begin{aligned} &3A_{333}(v^3)^2 + 3A_{113}(v^1)^2 \\ &+3A_{223}(v^2)^2 + 6A_{133} v^1 v^3 \\ &+6A_{233} v^2 v^3 + 6A_{334} v^3 v^4 \\ &+3A_{344}(v^4)^2 + 6A_{123} v^1 v^2 \\ &+6A_{134} v^1 v^4 + 6A_{234} v^2 v^4 \end{aligned} \right]$$

$$\frac{\partial}{\partial v^3} A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3} = 3 A_{i_1 i_2 3} v^{i_1} v^{i_2}$$

$$\begin{aligned}
A_{i_1 i_2 3} v^{i_1} v^{i_2} = \Big[\, & A_{113} (v^1)^2 + A_{123} v^1 v^2 + A_{133} v^1 v^3 + A_{134} v^1 v^4 \\
& + A_{213} v^1 v^2 + A_{223} (v^2)^2 + A_{233} v^2 v^3 + A_{234} v^2 v^4 \\
& + A_{313} v^1 v^3 + A_{323} v^2 v^3 + A_{333} (v^3)^2 + A_{334} v^3 v^4 \\
& + A_{413} v^1 v^4 + A_{423} v^2 v^4 + A_{433} v^3 v^4 + A_{344} (v^4)^2
\end{aligned}$$

$$\begin{aligned}
A_{i_1 i_2 3} v^{i_1} v^{i_2} = \Big[\, & A_{113} (v^1)^2 + 2 A_{123} v^1 v^2 + 2 A_{133} v^1 v^3 \\
& + 2 A_{134} v^1 v^4 + A_{223} (v^2)^2 \\
& + 2 A_{233} v^2 v^3 + 2 A_{234} v^2 v^4 \\
& + A_{333} (v^3)^2 + 2 A_{334} v^3 v^4 + A_{344} (v^4)^2
\end{aligned}$$

$$\begin{aligned}
3 A_{i_1 i_2 3} v^{i_1} v^{i_2} = \Big[\, & + 3 A_{333} (v^3)^2 \\
& 3 A_{113} (v^1)^2 + 3 A_{223} (v^2)^2 + 3 A_{344} (v^4)^2 \\
& + 6 A_{133} v^1 v^3 + 6 A_{233} v^2 v^3 \\
& + 6 A_{334} v^3 v^4 \\
& + 6 A_{123} v^1 v^2 + 6 A_{134} v^1 v^4 \\
& + 6 A_{234} v^2 v^4 \Big]
\end{aligned}$$

$$\boxed{\frac{\partial}{\partial v^{i_3}} A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3} = 3 A_{i_1 i_2 i_3} v^{i_1} v^{i_2}}$$

$$\boxed{\frac{\partial}{\partial v^{i_1}} A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3} = 3 A_{i_1 i_2 i_3} v^{i_2} v^{i_3}}$$

$$\boxed{\begin{aligned}&\frac{\partial}{\partial v^{i_1}} A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} = \\ &n A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_2} v^{i_3} v^{i_4} \cdots v^{i_{n-1}} v^{i_n}\end{aligned}}$$

$$\delta\left[A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} ds\right] =$$

$$\frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-1} i_n}}{\partial x^j} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} \delta x^j ds$$

$$+ n A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} d(\delta x^{i_1})$$

$$- n A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} \Delta_{i_{n+1} j} v^{i_{n+1}} d(\delta x^j)$$

$$+ A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} \Delta_{i_{n+1} j} v^{i_{n+1}} d(\delta x^j)$$

$$\delta\left[A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} ds\right] =$$

$$\frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-1} i_n}}{\partial x^j} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} \delta x^j ds$$

$$+ n A_{i_1 i_2 i_3 \cdots i_{n-1} j} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} d(\delta x^j)$$

$$-(n-1) A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} \Delta_{i_{n+1} j} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} v^{i_{n+1}} d(\delta x^j)$$

$$\delta \int A_{i_1 i_2 i_3 \ldots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \ldots v^{i_{n-1}} v^{i_n} \, ds =$$

$$\int \left[\begin{array}{l} \dfrac{\partial A_{i_1 i_2 i_3 \ldots i_{n-1} i_n}}{\partial x^j} v^{i_1} v^{i_2} v^{i_3} \ldots v^{i_{n-1}} v^{i_n} \\ - n \dfrac{d}{ds} \left\langle A_{i_1 i_2 i_3 \ldots i_{n-1} j} v^{i_1} v^{i_2} v^{i_3} \ldots v^{i_{n-1}} \right\rangle \\ + (n-1) \dfrac{d}{ds} \left\langle A_{i_1 i_2 i_3 \ldots i_{n-1} i_n} \Delta_{i_{n+1} j} v^{i_1} v^{i_2} v^{i_3} \ldots v^{i_{n-1}} v^{i_n} v^{i_{n+1}} \right\rangle \\ - \end{array} \right] (\delta x^j) \, ds$$

$$\frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-1} i_n}}{\partial x^j} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n}$$ grado n

$$- n \frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-1} j}}{\partial x^{i_n}} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n}$$ grado n

$$- n(n-1) A_{i_1 i_2 i_3 \cdots i_{n-1} j} a^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}}$$ (†) grado n

$$+ (n-1) \frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-1} i_n}}{\partial x^{i_{n+1}}} \Delta_{i_{n+2} j} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} v^{i_{n+1}} v^{i_{n+2}}$$ (*) grado $n+2$

$$+ n(n-1) A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} \Delta_{i_{n+1} j} a^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} v^{i_{n+1}}$$ (☆) grado $n+2$

$$+ (n-1) A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} \Delta_{i_{n+1} j} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} a^{i_{n+1}}$$ grado $n+2$ △

(+)

$$A_{i_1 i_2 i_3 \cdots i_{n-1} j}\, a^{i_1} v^{i_2} v^{i_3} v^{i_4} \cdots v^{i_{n-1}}$$

$$A_{i_1 i_2 i_3 \cdots i_{n-1} j}\, \Delta^{i_1 k} \left(\frac{\partial h_{kl}}{\partial x^m} - \frac{\partial h_{lm}}{\partial x^k} \right) v^l v^m v^{i_2} v^{i_3} v^{i_4} \cdots v^{i_{n-1}}$$

$$\cancel{A_{i_1 i_2 i_3 \cdots i_{n-1} j}\, \Delta^{i_{n-1} k} \left(\frac{\partial h_k}{} - \frac{\partial h}{\partial x^k} \right) v^{i_1} v^{i_2} v^{i_3} \cdots v^{i}}$$

$$\cancel{A_{i_1 i_2 i_3 \cdots i_{n-2} j}}$$

$$A_{i_1 i_2 i_3 \cdots i_{n-2} p j}\, \Delta^{pk} \left(\frac{\partial h_{k i_{n-1}}}{\partial x^{i_n}} \quad \frac{\partial h_{i_{n-1} i_n}}{\partial x^k} \right) v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-2}} v^{i_{n-1}} v^{i_n}$$

$$\boxed{\begin{gathered} n(n-1) A_{i_1 i_2 i_3 \cdots i_{n-1} j}\, a^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} = \\ n(n-1) A_{i_1 i_2 i_3 \cdots i_{n-2} p j}\, \Delta^{pk} \left(\frac{\partial h_{k i_{n-1}}}{\partial x^{i_n}} - \frac{\partial h_{i_{n-1} i_n}}{\partial x^k} \right) v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} \end{gathered}}$$

$\circledast$

$$\boxed{(n-3)\,\frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-2}}}{\partial X^{i_{n-1}}}\,\Delta^{\cdot}_{i_n j\,\cdot}\, v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n}}$$

$$A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} \Delta_{i_{n+1} j} \Delta^{i_1 p} \left(\frac{\partial h_{pq}}{\partial x^r} - \frac{\partial h_{qr}}{\partial x^p} \right) v^q v^r v^{i_2} v^{i_3} v^{i_4} \cdots v^{i_{n-1}} v^{i_n} v^{i_{n+1}}$$

$$A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} \Delta^{i_1 p} \left(\frac{\partial h_{pq}}{\partial x^r} - \frac{\partial h_{qr}}{\partial x^p} \right) v^q v^r v^{i_2} v^{i_3} v^{i_4} \cdots v^{i_{n-1}} v^{i_n} v_j$$

$$A_{i_1 i_2 i_3 \cdots i_{n-1} q} \Delta^{qp} \left(\frac{\partial h_{p i_n}}{\partial x^{i_{n+1}}} - \frac{\partial h_{i_n i_{n+1}}}{\partial x^p} \right) v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} v^{i_{n+1}} v_j$$

$$A_{i_1 i_2 i_3 \cdots i_{n-1} q} \Delta^{pq} \left(\frac{\partial h_{p i_n}}{\partial x^{i_{n+1}}} - \frac{\partial h_{i_n i_{n+1}}}{\partial x^p} \right) \Delta_{j i_{n+2}} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} v^{i_{n+1}} v^{i_{n+2}}$$

$$\boxed{(n-2)(n-3) A_{i_1 i_2 i_3 \cdots i_{n-3} q} \Delta^{pq} \Delta_{j i_n} \left(\frac{\partial h_{p i_{n-2}}}{\partial x^{i_{n-1}}} - \frac{\partial h_{i_{n-2} i_{n-1}}}{\partial x^p} \right) v^{i_1} v^{i_2} \cdots v^{i_{n-1}} v^{i_n}}$$

$$A_{i_1 i_2 i_3 \cdots\cdots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \cdots\cdots v^{i_n} a_j$$

$$A_{i_1 i_2 i_3 \cdots\cdots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \cdots\cdots v^{i_n} \left(\frac{\partial h_{j i_{n+1}}}{\partial x^{i_{n+2}}} - \frac{\partial h_{i_{n+1} i_{n+2}}}{\partial x^j} \right) v^{i_{n+1}} v^{i_{n+2}}$$

$$A_{i_1 i_2 i_3 \cdots\cdots i_{n-1} i_n} \left(\frac{\partial h_{j i_{n+1}}}{\partial x^{i_{n+2}}} - \frac{\partial h_{i_{n+1} i_{n+2}}}{\partial x^j} \right) v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_n} v^{i_{n+1}} v^{i_{n+2}}$$

$$\boxed{(n-3) A_{i_1 i_2 i_3 \cdots\cdots i_{n-2}} \left(\frac{\partial h_{j i_{n-1}}}{\partial x^{i_n}} - \frac{\partial h_{i_{n-1} i_n}}{\partial x^j} \right) v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_n}}$$

$$\left.\begin{aligned}
&\frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-1} i_n}}{\partial x^j} - n \frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-1} j}}{\partial x^{i_n}} \\
&- n(n-1) A_{i_1 i_2 i_3 \cdots i_{n-2} pj} \Delta^{pk} \left(\frac{\partial h_{k i_{n-1}}}{\partial x^{i_n}} - \frac{\partial h_{i_{n-1} i_n}}{\partial x^k} \right) \\
&+ (n-3) \frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-2}}}{\partial x^{i_{n-1}}} \Delta_{i_n j} \\
&+ (n-2)(n-3) A_{i_1 i_2 i_3 \cdots i_{n-3} q} \Delta^{pq} \Delta_{j i_n} \left(\frac{\partial h_{p i_{n-2}}}{\partial x^{i_{n-1}}} - \frac{\partial h_{i_{n-2} i_{n-1}}}{\partial x^p} \right) \\
&+ (n-3) A_{i_1 i_2 i_3 \cdots i_{n-2}} \left(\frac{\partial h_{j i_{n-1}}}{\partial x^{i_n}} - \frac{\partial h_{i_{n-1} i_n}}{\partial x^j} \right)
\end{aligned}\right\} = 0$$

$$\mathcal{L}\,ds = A\,ds + A_{ij}\,v^i dx^j + A_{ijkl}\,v^i v^j v^k dx^l + \ldots$$

$$\underline{\delta\{\mathcal{L}\,ds\}}$$

$$\frac{\partial A}{\partial x^p}\delta x^p ds + A\,\Delta_{ip}\,v^i d(\delta x^p)$$

$$+ \frac{\partial A_{ij}}{\partial x^p} v^i v^j \delta x^p ds \,.$$

$$\mathcal{L}\, ds = \begin{cases} A\, ds + A_i\, dx^i + A_{ij}\, v^i dx^j + A_{ijk}\, v^i v^j dx^k \\ + A_{ijkl}\, v^i v^j v^k dx^l + \ldots\ldots \end{cases}$$

$$\underline{\delta\{\mathcal{L}\, ds\}}$$

$$\frac{\partial A}{\partial x^p}\delta x^p ds + \cancel{\frac{\partial A_i}{\partial x^p} v^i \delta x^p ds} + A\, \Delta_{ip}\, v^i\, d(\delta x^p)$$

$$+ \frac{\partial A_i}{\partial x^p} v^i \delta x^p ds + A_p\, d(\delta x^p)$$

$$+ \frac{\partial A_{ij}}{\partial x^p} v^i v^j \delta x^p ds + A_{pj}\, v^j\, d(\delta x^p) + A_{ip}\, v^i\, d(\delta x^p)$$

$$+ \frac{\partial A_{ijk}}{\partial x^p} v^i v^j v^k \delta x^p ds + 2 A_{ijk}\, v^i$$

$$\mathcal{L}_0\, ds = A\, ds$$

$$\underline{\delta A ds.}$$

$$\frac{\partial A}{\partial x^p}\delta x^p ds + A\,\Delta_{ip} v^i d(\delta x^p)$$

$$\frac{\partial A}{\partial x^p} - \frac{\partial A}{\partial x^j}\Delta_{ip} v^i v^j - A\, a_p$$

$$\frac{\partial A}{\partial x^p} - \frac{\partial A}{\partial x^j}\Delta_{ip} v^i v^j - A\left(\frac{\partial h_{pi}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^p}\right) v^i v^j$$

$$\frac{\partial A}{\partial x^p} - \left[\frac{\partial A}{\partial x^j}\Delta_{ip} - A\frac{\partial h_{pi}}{\partial x^j} + A\frac{\partial h_{ij}}{\partial x^p}\right] v^i v^j$$

$$\mathcal{L}_1\, ds = A_i\, dx^i$$

$$\delta \mathcal{L}_1\, ds = \delta A_i\, dx^i$$

$$\mathcal{L}_1\, ds = A_i v^i ds \qquad \boxed{\mathcal{L}_1 = A_i v^i}$$

$$\mathcal{L}_1\, ds = A_i\, dx^i$$

$$\underline{\delta A_i\, dx^i}$$

$$\frac{\partial A_i}{\partial x^p} v^i \delta x^p ds + A_p d(\delta x^p)$$

$$\boxed{\frac{\partial A_i}{\partial x^p} v^i \delta x^p ds + A_p\, d(\delta x^p)}$$

$$\boxed{\delta A_i \, dx^i}$$

$$\boxed{\frac{\partial A_i}{\partial x^p} v^i \delta x^p ds + A_p d(\delta x^p)}$$

$$\frac{\partial A_i}{\partial x^p} v^i - \frac{\partial A_p}{\partial x^i} v^i$$

$$\left[\frac{\partial A_i}{\partial x^p} - \frac{\partial A_p}{\partial x^i}\right] v^i$$

$$\mathcal{L}_2 ds = A_{ij} v^i dx^j = A_{ij} v^i v^j ds$$

$$\mathcal{L}_2 = A_{ij} v^i v^j$$

$$\delta A_{ij} v^i dx^j$$

$$\frac{\partial A_{ij}}{\partial x^p} v^i v^j \delta x^p ds +$$

$$+ A_{pj} v^j d(\delta x^p) - A_{ij} v^i v^j \Delta_{pq} v^q d(\delta x^p)$$

$$+ A_{ip} v^i d(\delta x^p)$$

$$\delta A_{ij} v^i dx^j$$

$$\frac{\partial A_{ij}}{\partial x^p} v^i v^j \delta x^p ds +$$

$$+ 2 A_{ip} v^i d(\delta x^p) - A_{ij} \Delta_{pq} v^i v^j v^q d(\delta x^p)$$

$$\frac{\partial A_{ij}}{\partial x^p} v^i v^j - 2\frac{\partial A_{ip}}{\partial x^j} v^i v^j - 2A_{ip}\, a^i$$

$$+ \frac{\partial A_{ij}}{\partial x^l} \Delta_{pk}\, v^i v^j v^k v^l + 2A_{ij}\, \Delta_{pk}\, a^i v^j v^k$$

$$+ A_{ij}\, \Delta_{pk}\, v^i v^j a^k$$

$$\left[\frac{\partial A_{ij}}{\partial x^p} - 2\frac{\partial A_{ip}}{\partial x^j} - 2A_{pq}\Delta^{qm}\left(\frac{\partial h_{mi}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^m}\right)\right] v^i v^j$$

$$+ \left[\frac{\partial A_{ij}}{\partial x^l}\Delta_{pk} + 2A_{jq}\Delta_{pk}\Delta^{qm}\left(\frac{\partial h_{mi}}{\partial x^l} - \frac{\partial h_{il}}{\partial x^m}\right) + A_{ij}\left(\frac{\partial h_{pk}}{\partial x^l} - \frac{\partial h_{kl}}{\partial x^p}\right)\right] v^i v^j v^k v^l$$

Introducción de una métrica completamente euclidiana (cartesiana).

$$\cancel{ds^2 = dx^2 + dy^2 - dz^2}$$

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

$$-d\sigma^2 = -d\tau^2 - dx^2 - dy^2 - dz^2$$

$$\boxed{\tau = i t} \qquad \boxed{\sigma = i s} \qquad \boxed{ds^2 = -d\sigma^2}$$

$$d\sigma^2 = d\tau^2 + dx^2 + dy^2 + dz^2$$

$$\tau \equiv x^1;\ x \equiv x^2;\ y \equiv x^3;\ z \equiv x^4$$

$$\boxed{d\sigma^2 = \delta_{ij}\, dx^i dx^j}$$

$$t \equiv X^1;\ x \equiv X^2;\ y \equiv X^3;\ z \equiv X^4$$

$$\boxed{i X^1 = x^1}$$

$$\frac{d^2 X^1}{ds^2} = \Delta^{1p}\left[\frac{\partial h_{pj}}{\partial X^k} - \frac{\partial h_{jk}}{\partial X^p}\right]\frac{dX^j}{ds}\frac{dX^k}{ds}$$

$$-i\frac{d^2 X^1}{d\sigma^2} = i\left[\frac{\partial h_{1j}}{\partial X^k} - \frac{\partial h_{jk}}{\partial X^1}\right]\frac{dX^j}{ds}\frac{dX^k}{ds}$$

$$\frac{d^2x^1}{d\sigma^2} = i\frac{\partial h_{1j}}{\partial X^k}\frac{dX^j}{d\sigma}\frac{dX^k}{d\sigma} + \frac{\partial h_{jk}}{\partial x^1}\frac{dX^j}{d\sigma}\frac{dX^k}{d\sigma}$$

$$\frac{d^2x^\alpha}{ds^2} = \Delta^{\alpha p}\left[\frac{\partial h_{pj}}{\partial X^k} - \frac{\partial h_{jk}}{\partial X^p}\right]\frac{dX^j}{ds}\frac{dX^k}{ds}$$

$$\frac{d^2x^\alpha}{d\sigma^2} = -\frac{\partial h_{\alpha j}}{\partial x^k}\frac{dX^j}{d\sigma}\frac{dx^k}{d\sigma} + \frac{\partial h_{jk}}{\partial x^\alpha}\frac{dX^j}{d\sigma}\frac{dX^k}{d\sigma}$$

$$\boxed{\begin{aligned} x^1 &= iX^1 \\ x^2 &= X^2 \\ x^3 &= X^3 \\ x^4 &= X^4 \end{aligned}}$$

$$x = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} X$$

$$M = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad M^{-1} = \begin{pmatrix} -i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\cancel{h_{ij} = \bar{M}_i^{\,p} M_j^{\,q} H_{pq}}$$

$$h_{ij} = M^{-1}{}_i^{\,p}\, M^{-1}{}_j^{\,q}\, H_{pq}$$

$$v^i = \frac{\partial x^i}{\partial X^j} V^j$$

$$v_i = \frac{\partial X^j}{\partial x^i} V_j$$

$$h_{ij} = \frac{\partial X^p}{\partial x^i}\frac{\partial X^q}{\partial x^j} H_{pq}$$

$$h_{11} = - H_{11}$$

$$h_{1\alpha} = \frac{\partial X^1}{\partial x^1}\frac{\partial X^{\alpha}}{\partial x^{\alpha}} H_{1\alpha}$$

$$h_{1\alpha} = \frac{1}{i} H_{1\alpha} = - i H_{1\alpha}$$

$$\boxed{h_{1\alpha} = - i H_{1\alpha}} \quad \boxed{h_{11} = - H_{11}}$$

$$\boxed{h_{\alpha\beta} = H_{\alpha\beta}}$$

$$\frac{d^2 X^1}{ds^2} = \left(\frac{\partial h_{1j}}{\partial X^k}\right)\frac{dX^j}{ds}\frac{dX^k}{ds} - \frac{\partial h_{jk}}{\partial X^1}\frac{dX^j}{ds}\frac{dX^k}{ds}$$

$$\frac{d^2 X^1}{d\sigma^2} = \Bigg[\frac{\partial h_{11}}{\partial X^1}\frac{dX^1}{d\sigma}\frac{dX^1}{d\sigma} + \frac{\partial h_{11}}{\partial X^{\alpha}}\frac{dX^1}{d\sigma}\frac{dX^{\alpha}}{d\sigma}$$

$$+ \frac{\partial h_{1\alpha}}{\partial X^1}\frac{dX^{\alpha}}{d\sigma}\frac{dX^1}{d\sigma} + \frac{\partial h_{1\alpha}}{\partial X^{\beta}}\frac{dX^{\alpha}}{d\sigma}\frac{dX^{\beta}}{d\sigma}$$

$$-$$

Resultados

En la transformación:

$$x^1 = i X^1 ; \quad x^2 = X^2 ; \quad x^3 = X^3 ; \quad x^4 = X^4$$

el tensor H_{ij} se transforma como sigue:

$$\boxed{\begin{aligned} h_{11} &= - H_{11} \\ h_{1\alpha} &= - i\, H_{1\alpha} \\ h_{\alpha 1} &= h_{1\alpha} \\ h_{\alpha\beta} &= H_{\alpha\beta} \end{aligned} \qquad \alpha, \beta = 2, 3, 4.}$$

$$dS^2 = (dX^1)^2 - (dX^2)^2 - (dX^3)^2 - (dX^4)^2$$

$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + (dx^4)^2$$

$$\boxed{dS^2 = - ds^2}$$

$$\boxed{\cancel{\sigma = i s}} \qquad \boxed{S = i s}$$

Transformación de las ecuaciones:

$$\frac{d^2\bar{X}^i}{d\bar{S}^2} = \Delta^{ip}\left[\frac{\partial H_{pj}}{\partial \bar{X}^k} - \frac{\partial H_{jk}}{\partial \bar{X}^p}\right]\frac{d\bar{X}^j}{d\bar{S}}\frac{d\bar{X}^k}{d\bar{S}}$$

$$\frac{d^2\bar{X}^1}{d\bar{S}^2} = \left(\frac{\partial H_{1j}}{\partial \bar{X}^k} - \frac{\partial H_{jk}}{\partial \bar{X}^1}\right)\frac{d\bar{X}^j}{d\bar{S}}\frac{d\bar{X}^k}{d\bar{S}}$$

$$i\frac{d^2\bar{X}^1}{ds^2} = \begin{cases} i\dfrac{\partial H_{11}}{\partial \bar{X}^1}\dfrac{d\bar{X}^1}{ds}\dfrac{d\bar{X}^1}{ds} + i\dfrac{\partial H_{1\alpha}}{\partial \bar{X}^1}\dfrac{d\bar{X}^\alpha}{ds}\dfrac{d\bar{X}^1}{ds} \\ + i\dfrac{\partial H_{11}}{\partial \bar{X}^\alpha}\dfrac{d\bar{X}^1}{ds}\dfrac{d\bar{X}^\alpha}{ds} + i\dfrac{\partial H_{1\alpha}}{\partial \bar{X}^\beta}\dfrac{d\bar{X}^\alpha}{ds}\dfrac{d\bar{X}^\beta}{ds} \\ - i\dfrac{\partial H_{11}}{\partial \bar{X}^1}\dfrac{d\bar{X}^1}{ds}\dfrac{d\bar{X}^1}{ds} - i\dfrac{\partial H_{1\alpha}}{\partial \bar{X}^1}\dfrac{d\bar{X}^\alpha}{ds}\dfrac{d\bar{X}^1}{ds} \leftarrow \text{uno} \\ - i\dfrac{\partial H_{\alpha 1}}{\partial \bar{X}^1}\dfrac{d\bar{X}^1}{ds}\dfrac{d\bar{X}^\alpha}{ds} - i\dfrac{\partial H_{\alpha\beta}}{\partial \bar{X}^1}\dfrac{d\bar{X}^\alpha}{ds}\dfrac{d\bar{X}^\beta}{ds} \end{cases}$$

$$\frac{d^2x^1}{ds^2} = \begin{cases} -\dfrac{\partial h_{11}}{\partial x^1}\dfrac{dx^1}{ds}\dfrac{dx^1}{ds} - \dfrac{\partial h_{1\alpha}}{\partial x^1}\dfrac{dx^\alpha}{ds}\dfrac{dx^1}{ds} \\ -\dfrac{\partial h_{11}}{\partial x^\alpha}\dfrac{dx^1}{ds}\dfrac{dx^\alpha}{ds} - \dfrac{\partial h_{1\alpha}}{\partial x^\beta}\dfrac{dx^\alpha}{ds}\dfrac{dx^\beta}{ds} \\ +\dfrac{\partial h_{11}}{\partial x^1}\dfrac{dx^1}{ds}\dfrac{dx^1}{ds} + \dfrac{\partial h_{1\alpha}}{\partial x^1}\dfrac{dx^\alpha}{ds}\dfrac{dx^1}{ds} \\ +\dfrac{\partial h_{\alpha 1}}{\partial x^1}\dfrac{dx^1}{ds}\dfrac{dx^\alpha}{ds} + \dfrac{\partial h_{\alpha\beta}}{\partial x^1}\dfrac{dx^\alpha}{ds}\dfrac{dx^\beta}{ds} \end{cases}$$

$$\boxed{\frac{d^2x^1}{ds^2} = -\left(\frac{\partial h_{1j}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^1}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}.}$$

$$\frac{d^2X^\alpha}{ds^2} = -\left(\frac{\partial H_{\alpha \overset{\text{jota}}{j}}}{\partial X^k} - \frac{\partial H_{jk}}{\partial X^\alpha}\right)\frac{dX^j}{ds}\frac{dX^k}{ds}$$

$$\begin{aligned}\frac{d^2X^\alpha}{ds^2} = \Big[& -\frac{\partial H_{\alpha 1}}{\partial X^1}\frac{dX^1}{ds}\frac{dX^1}{ds} - \frac{\partial H_{\alpha\beta}}{\partial X^1}\frac{dX^\beta}{ds}\frac{dX^1}{ds} \\ & -\frac{\partial H_{\alpha 1}}{\partial X^\beta}\frac{dX^1}{ds}\frac{dX^\beta}{ds} - \frac{\partial H_{\alpha\beta}}{\partial X^\gamma}\frac{dX^\beta}{ds}\frac{dX^\gamma}{ds} \\ & +\frac{\partial H_{11}}{\partial X^\alpha}\frac{dX^1}{ds}\frac{dX^1}{ds} + \frac{\partial H_{1\beta}}{\partial X^\alpha}\frac{dX^1}{ds}\frac{dX^\beta}{ds} \\ & +\frac{\partial H_{\beta 1}}{\partial X^\alpha}\frac{dX^\beta}{ds}\frac{dX^1}{ds} + \frac{\partial H_{\beta\gamma}}{\partial X^\alpha}\frac{dX^\beta}{ds}\frac{dX^\gamma}{ds}\Big]\end{aligned}$$

$$\begin{aligned}\frac{d^2x^\alpha}{ds^2} = \Big[& -\frac{\partial h_{\alpha 1}}{\partial x^1}\frac{dx^1}{ds}\frac{dx^1}{ds} - \frac{\partial h_{\alpha\beta}}{\partial x^1}\frac{dx^\beta}{ds}\frac{dx^1}{ds} \\ & -\frac{\partial h_{\alpha 1}}{\partial x^\beta}\frac{dx^1}{ds}\frac{dx^\beta}{ds} - \frac{\partial h_{\alpha\beta}}{\partial x^\gamma}\frac{dx^\beta}{ds}\frac{dx^\gamma}{ds} \\ & +\frac{\partial h_{11}}{\partial x^\alpha}\frac{dx^1}{ds}\frac{dx^1}{ds} + \frac{\partial h_{1\beta}}{\partial x^\alpha}\frac{dx^1}{ds}\frac{dx^\beta}{ds} \\ & +\frac{\partial h_{\beta 1}}{\partial x^\alpha}\frac{dx^\beta}{ds}\frac{dx^1}{ds} + \frac{\partial h_{\beta\gamma}}{\partial x^\alpha}\frac{dx^\beta}{ds}\frac{dx^\gamma}{ds}\Big]\end{aligned}$$

$$\frac{d^2x^\alpha}{ds^2} = -\left(\frac{\partial h_{\alpha j}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^\alpha}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}.$$

Resultados def

Métrica pseudoeuclidiana.
Coordenadas reales
$dS^2 > 0$ intervalos temporaloides.

$$dS^2 = (dX^1)^2 - (dX^2)^2 - (dX^3)^2 - (dX^4)^2$$

$$dS^2 = \Delta_{ij}\, dX^i\, dX^j$$

Movimiento de una partícula en el campo gravitacional

$$\frac{d^2X^i}{dS^2} = \Delta^{ip}\left(\frac{\partial H_{pj}}{\partial X^k} - \frac{\partial H_{jk}}{\partial X^p}\right)\frac{dX^j}{dS}\frac{dX^k}{dS}$$

Rel

iX^1

X^2

X^3

X^4

iS

$dS^2 =$

H_{11}

$iH_{1\alpha}$

$iH_{\alpha 1}$

$-H_{\alpha\beta}$

definitivos.

Métrica euclidiana completa.
(completamente euclidiana).
coordenada temporal imaginaria.
$ds^2 > 0$ intervalo espacialoide.

$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + (dx^4)^2$$

$$ds^2 = \delta_{ij}\, dx^i dx^j$$

Movimiento de una partícula en el campo gravitacional.

$$\frac{d^2x^i}{ds^2} = \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}.$$

Relaciones.

$iX^1 = x^1$

$X^2 = x^2$

$X^3 = x^3$

$X^4 = x^4$

$iS = s$

$dS'^2 = -ds^2$

$H_{11} = h_{11}$

$iH_{1\alpha} = h_{1\alpha}$

$iH_{\alpha 1} = h_{\alpha 1}$

$-H_{\alpha\beta} = h_{\alpha\beta}$

$$\delta(ds)$$

$$ds^2 = dx^i\, dx^i$$

$$\delta(ds)^2 = 2\, ds\, \delta(ds) = 2\, dx^i\, d(\delta x^i)$$

$$\boxed{\delta\, ds = v^i\, d(\delta x^i)} \longleftarrow$$

$$\delta v^i = \delta \frac{dx^i}{ds} = \frac{ds\, d(\delta x^i) - dx^i\, v^j d(\delta x^j)}{ds^2}$$

$$\boxed{\delta v^i = \frac{d}{ds}(\delta x^i) - v^i\left[v^j\, d(\delta x^j)\right]} \longleftarrow \longleftarrow$$

$$\mathcal{L}_0\, ds = A\, ds \qquad \mathcal{L}_0 = A$$

$$\delta(A\, ds)$$

$$\frac{\partial A}{\partial x^p}\delta x^p\, ds + A\, v^i\, d(\delta x^i)$$

$$\boxed{\delta \int A\, ds}$$

$$\frac{\partial A}{\partial x^p}\delta x^p\, ds - \frac{\partial A}{\partial x^j} v^j v^p\, \delta x^p\, ds$$

$$-A\, a^p\, \delta x^p\, ds$$

$$\frac{\partial A}{\partial x^p} - A a^p - \frac{\partial A}{\partial x^j} v^j v^p$$

$$\frac{\partial A}{\partial x^p} - A\left(\frac{\partial h_{pi}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^p}\right) v^i v^j - \frac{\partial A}{\partial x^j} v^j v^p$$

$$\boxed{\left[\frac{\partial A}{\partial x^p}\delta_{ij} - A\frac{\partial h_{pi}}{\partial x^j} + A\frac{\partial h_{ij}}{\partial x^p} - \frac{\partial A}{\partial x^j}\delta_{pi}\right] v^i v^j}$$

$$\boxed{\delta \int A\,ds = \int \left[\frac{\partial A}{\partial x^p}\delta_{ij} - A\frac{\partial h_{pi}}{\partial x^j} + A\frac{\partial h_{ij}}{\partial x^p} - \frac{\partial A}{\partial x^j}\delta_{pi}\right] v^i v^j \delta x^p ds}$$

$$\mathcal{L}_2\, ds = A_{ij}\, v^i\, dx^j$$

$$\delta \mathcal{L}_2\, ds = \left[\frac{\partial A_{ij}}{\partial x^p} v^i v^j \delta x^p ds \right.$$
$$+ A_{pj}\, v^j\, d(\delta x^p)$$
$$- A_{ij}\, v^i v^j\, v^p\, d(\delta x^p)$$
$$\left. \left[+ A_{ip}\, v^i\, d(\delta x^p) \right. \right]$$

$$\boxed{\delta \mathcal{L}_2\, ds -}$$

$$\delta \int \mathcal{L}_2\, ds$$

$$\int \left[\frac{\partial A_{ij}}{\partial x^p} v^i v^j - \frac{\partial A_{pj}}{\partial x^i} v^i v^j - A_{pj}\, a^j \right.$$
$$+ \frac{\partial A_{ij}}{\partial x^k} v^i v^j v^k v^p + 2 A_{ij}\, a^i v^j v^p$$
$$+ A_{ij}\, v^i v^j a^p - \frac{\partial A_{ip}}{\partial x^j} v^i v^j$$
$$\left. - A_{ip}\, a^i \right] \delta x^p ds$$

$$\left[\frac{\partial A_{ij}}{\partial x^p} - \frac{\partial A_{pj}}{\partial x^i} - A_{pk}\left(\frac{\partial h_{ki}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^k}\right) - \frac{\partial A_{ip}}{\partial x^j} - A_{kp}\left(\frac{\partial h_{ki}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^k}\right)\right] v^i v^j$$

$$\left.\begin{array}{l} \frac{\partial A}{\partial x^p}\delta_{ij} - A\frac{\partial h_{pi}}{\partial x^j} + A\frac{\partial h_{ij}}{\partial x^p} - \frac{\partial A}{\partial x^j}\delta_{pi} \\ + \frac{\partial A_{ij}}{\partial x^p} - \frac{\partial A_{pj}}{\partial x^i} - 2A_{pk}\left(\frac{\partial h_{ki}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^k}\right) \\ - \frac{\partial A_{ip}}{\partial x^j} \end{array}\right\} = 0$$

$$\left(\frac{\partial A_{ij}}{\partial x^k} + 2A_{qj}\left(\frac{\partial h_{qi}}{\partial x^k} - \frac{\partial h_{ik}}{\partial x^q}\right)\right) \cancel{+ A_{ij}\frac{\partial h_{pk}}{\partial x^l}}\ v^i v^j v^k v^p$$

$$+ A_{ij} v^i v^j \left(\frac{\partial h_{pk}}{\partial x^l} - \frac{\partial h_{kl}}{\partial x^p}\right) v^k v^l$$

$$\left.\begin{array}{l} \frac{\partial A_{ij}}{\partial x^k}\delta_{pl} + 2A_{qj}\delta_{pl}\left(\frac{\partial h_{qi}}{\partial x^k} - \frac{\partial h_{ik}}{\partial x^q}\right) \\ + A_{ij}\left(\frac{\partial h_{pk}}{\partial x^l} - \frac{\partial h_{kl}}{\partial x^p}\right) \end{array}\right\} v^i v^j v^k v^l$$

Caso del espacio físico unidimensional

$$\frac{d^2x^1}{ds^2} = \left(\frac{\partial h_{1j}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^1}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}$$

$$\frac{d^2x^1}{ds^2} = \left[\left(\frac{\partial h_{11}}{\partial x^2} - \frac{\partial h_{12}}{\partial x^1}\right)\frac{dx^1}{ds}\frac{dx^2}{ds} + \left(\frac{\partial h_{12}}{\partial x^1} - \frac{\partial h_{21}}{\partial x^1}\right)\frac{dx^2}{ds}\frac{dx^1}{ds} + \left(\frac{\partial h_{12}}{\partial x^2} - \frac{\partial h_{22}}{\partial x^1}\right)\frac{dx^2}{ds}\frac{dx^2}{ds}\right]$$

$$\frac{d^2x^1}{ds^2} = \left[\left(\frac{\partial h_{11}}{\partial x^2} - \frac{\partial h_{12}}{\partial x^1}\right)\frac{dx^1}{ds}\frac{dx^2}{ds} + \left(\frac{\partial h_{12}}{\partial x^2}\cancel{\frac{dx^2}{d}} - \frac{\partial h_{22}}{\partial x^1}\right)\frac{dx^2}{ds}\frac{dx^2}{ds}\right]$$

$$x^1 \equiv \tau \quad ; \quad x^2 \equiv x$$

$$\frac{d^2\tau}{ds^2} = \left[\left(\frac{\partial h_{11}}{\partial x} - \frac{\partial h_{12}}{\partial \tau}\right)\frac{d\tau}{ds}\frac{dx}{ds} + \left(\frac{\partial h_{12}}{\partial x}\cancel{\frac{dx}{ds}}\cancel{\frac{dx}{ds}} - \frac{\partial h_{22}}{\partial \tau}\right)\left(\frac{dx}{ds}\right)^2\right.$$

La Integral de Acción

La integral de acción para el caso de un campo generado por una ~~partícula~~ punto masa en movimiento ~~arbitrario~~ con cuadrivector $\hat{V}$elocidad constante

μ ------ masa en gramos del punto masa

G ------ constante gravitacional (CGS)

$\hat{V}$ ------ cuadrivector velocidad del punto masa.

$\vec{V}$ ------ vector velocidad del punto masa.

V ------ velocidad del punto masa en cm/seg.

$$\hat{V} = \left(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}} , \frac{\frac{\vec{V}}{c}}{\sqrt{1-\frac{V^2}{c^2}}} \right)$$

$$\hat{V} \cdot \hat{V} = 1$$

Coordenadas

x ---- abscisa en cm.

y ---- ordenada en cm

z ---- cota en cm

t ---- tiempo en seg.

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

$$\boxed{ds = c\sqrt{1-\frac{v^2}{c^2}}\, dt}$$

(t, x, y, z) ---- acontecimiento en la línea de universo de la partícula exploradora

(T, X, Y, Z) ---- acontecimiento retardado en la línea de universo m ---- masa de la partícula exploradora en ~~[illegible]~~ gramos.

$\hat{v}$ ---- cuadrivector velocidad de la partícula exploradora

$\vec{v}$ ---- vector velocidad de la partícula exploradora

v ---- velocidad de la partícula exploradora en cm/seg.

$$\hat{v} = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} , \frac{\frac{\vec{v}}{c}}{\sqrt{1-\frac{v^2}{c^2}}} \right)$$

$$\boxed{\begin{aligned} ds &= c\sqrt{1-\frac{v^2}{c^2}}\,dt \\ ds^2 &= c^2dt^2 - dx^2 - dy^2 - dz^2 \end{aligned}}$$

$$\hat{v} \cdot \hat{v} = 1 .$$

La integral de acción es:

$$\boxed{\mathcal{J} = \frac{1}{2} mc \int e^{\frac{2G\mu}{c^2(\hat{r}\cdot\hat{V})}} \left[2 - \frac{1}{(\hat{V}\cdot\hat{v})^2} \right] (\hat{V}\cdot\hat{v})\, ds}$$

$$\hat{V} = \left(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}} , \frac{\frac{\vec{V}}{c}}{\sqrt{1-\frac{V^2}{c^2}}} \right)$$

$$\hat{v} = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} , \frac{\frac{\vec{v}}{c}}{\sqrt{1-\frac{v^2}{c^2}}} \right)$$

$$\boxed{\hat{V}\cdot\hat{v} = \frac{1-\frac{\vec{V}\cdot\vec{v}}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{V^2}{c^2}}}}$$

$$\boxed{ds = c\sqrt{1-\frac{v^2}{c^2}}\,dt}$$

$$\left[2-\frac{1}{(\hat{V}\cdot\hat{v})^2}\right](\hat{V}\cdot\hat{v})\,ds =$$

$$= \left[2-\frac{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{V^2}{c^2}\right)}{\left(1-\frac{\vec{V}\cdot\vec{v}}{c^2}\right)^2}\right]\frac{\left(1-\frac{\vec{V}\cdot\vec{v}}{c^2}\right)\left[c\cancel{\sqrt{1-\frac{v^2}{c^2}}}\,dt\right]}{\cancel{\sqrt{1-\frac{v^2}{c^2}}}\sqrt{1-\frac{V^2}{c^2}}}$$

$$= \left[\frac{2\left(1-\frac{\vec{V}\cdot\vec{v}}{c^2}\right)}{\sqrt{1-\frac{V^2}{c^2}}} - \frac{\left(1-\frac{v^2}{c^2}\right)\sqrt{1-\frac{V^2}{c^2}}}{\left(1-\frac{\vec{V}\cdot\vec{v}}{c^2}\right)}\right]c\,dt$$

$$\boxed{\frac{1}{2}mc\left[2-\frac{1}{(\hat{V}\cdot\hat{v})^2}\right](\hat{V}\cdot\hat{v})\,ds = \frac{1}{2}mc^2\left[\frac{2\left(1-\frac{\vec{V}\cdot\vec{v}}{c^2}\right)}{\sqrt{1-\frac{V^2}{c^2}}} - \frac{\left(1-\frac{v^2}{c^2}\right)\sqrt{1-\frac{V^2}{c^2}}}{\left(1-\frac{\vec{V}\cdot\vec{v}}{c^2}\right)}\right]dt}$$

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$$\hat{r} = \left(c(t-T),\ x-X,\ y-Y,\ z-Z\right)$$

$$\hat{V} = \left[\frac{1}{\sqrt{1-\frac{V^2}{c^2}}},\ \frac{\frac{\vec{V}}{c}}{\sqrt{1-\frac{V^2}{c^2}}}\right]$$

$$\hat{r} = \left[c(t-T),\ \vec{r}\right]$$

$$(\hat{V}\cdot\hat{r}) = \frac{c(t-T) - \vec{r}\cdot\frac{\vec{V}}{c}}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$c(t-T) = r$$

$$\left(\hat{V}\cdot\hat{r}\right) = \frac{r - \frac{\vec{r}\cdot\vec{V}}{c}}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$\left(\hat{V}\cdot\hat{r}\right) = \frac{r - \frac{\vec{r}\cdot\vec{V}}{c}}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$\cancel{e^{\frac{2G\mu\sqrt{1-\frac{V^2}{c^2}}}{c^2\left(r-\frac{\vec{r}\cdot\vec{V}}{c}\right)}}}$$

$$\boxed{e^{\frac{2G\mu}{c^2\left(\hat{r}\cdot\hat{V}\right)}} = e^{\frac{2G\mu\sqrt{1-\frac{V^2}{c^2}}}{c^2\left(r-\frac{\vec{r}\cdot\vec{V}}{c}\right)}}}$$

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$$\gamma^{\mu} = \frac{1}{2} m c^2 \int e^{\frac{2G\mu\sqrt{1-\frac{V^2}{c^2}}}{c^2\left(r-\frac{\vec{r}\cdot\vec{V}}{c}\right)}} \left[\quad\right] dt$$

$$\boxed{\left[\frac{2\left(1-\frac{(\vec{V}\cdot\vec{v})}{c^2}\right)}{\sqrt{1-\frac{V^2}{c^2}}} - \frac{\left(1-\frac{v^2}{c^2}\right)\sqrt{1-\frac{V^2}{c^2}}}{\left(1-\frac{\vec{V}\cdot\vec{v}}{c^2}\right)}\right]}$$

Para simplificar la notación hago $c=1$ (provisionalmente).

$$\boxed{\frac{G\mu}{c^2} = M}$$

$$\boxed{\gamma^{\mu} = \frac{1}{2} m \int e^{\frac{2M\sqrt{1-V^2}}{r-\vec{r}\cdot\vec{V}}} \left[\frac{2\left[(\vec{V}\cdot\vec{v})\right]}{\sqrt{1-V^2}} - \frac{(1-v^2)\sqrt{1-V^2}}{(1-\vec{V}\cdot\vec{v})}\right] dt}$$

$$\boxed{\mathcal{R} = \frac{1}{r-\vec{r}\cdot\vec{V}}}$$

$$\boxed{\mathcal{I} = \frac{1}{2} m \int e^{2M\sqrt{1-V^2}\,\mathcal{R}} \left[\frac{2\left[1-(\vec{V}\cdot\vec{v})\right]}{\sqrt{1-V^2}} - \frac{(1-v^2)\sqrt{1-V^2}}{\left[1-(\vec{V}\cdot\vec{v})\right]} \right] dt}$$

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$$\boxed{\mathcal{L} = e^{2M\sqrt{1-V^2}\,\mathcal{R}} \left[\frac{2\left[1-(\vec{V}\cdot\vec{v})\right]}{\sqrt{1-V^2}} - \frac{(1-v^2)\sqrt{1-V^2}}{\left[1-(\vec{V}\cdot\vec{v})\right]} \right]}$$

Desarrollo en serie de $\mathcal{L}$ según potencias crecientes de c^{-1}

$$\sqrt{1-V^2} = \left[1 + \frac{1}{2}(-V^2) + \frac{(\frac{1}{2})(-\frac{1}{2})}{1\cdot 2}(-V^2)^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{1\cdot 2\cdot \cancel{3}}(-V^2)^3 + \cdots\right]$$

$$\boxed{\sqrt{1-V^2} = \left[1 - \frac{1}{2}V^2 - \frac{1}{8}V^4 - \frac{1}{16}V^6 + \cdots\right]}$$

$$\boxed{\left(\sqrt{1-V^2}\right)^2 = 1 - V^2}$$

$$\left(\sqrt{1-V^2}\right)^3 = \left[1 + \frac{3}{2}(-V^2) + \frac{(\frac{3}{2})(\frac{1}{2})}{1\cdot 2}(-V^2)^2 + \frac{(\frac{\cancel{3}}{2})(\frac{1}{2})(-\frac{1}{2})}{1\cdot 2\cdot \cancel{3}}(-V^2)^3 + \cdots\right]$$

$$\boxed{\left(\sqrt{1-V^2}\right)^3 = \left[1 - \frac{3}{2}V^2 + \frac{3}{8}V^4 + \frac{1}{8}V^6 + \cdots\right]}$$

$$e^{2M\sqrt{1-V^2}R} = \left[1 + 2M\sqrt{1-V^2}R + 2M^2(1-V^2)R^2 + \frac{4}{3}M^3\left(\sqrt{1-V^2}\right)^3 R^3\right]$$

$$\boxed{e^{2M\sqrt{1-V^2}R} = \left[\begin{array}{l} 1 \\ + 2MR - MV^2R - \frac{1}{4}MV^4R \\ + 2M^2R^2 - 2M^2V^2R^2 \\ + \frac{4}{3}M^3R^3 \end{array}\right]}$$

comprobado

$$\sqrt{1-V^2} = 1 - \frac{1}{2}V^2 - \frac{1}{8}V^4 - \frac{1}{16}V^6 + \dots\dots$$

$$2 - 2(\vec{V}\cdot\vec{v})$$

$$\cancel{\frac{2(1-\vec{V}\cdot\vec{v})}{\sqrt{1-V^2}} =}$$

$$(1-V^2)^{-\frac{1}{2}} = 1 + \left(-\tfrac{1}{2}\right)(-V^2) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1\cdot 2}(-V^2)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1\cdot 2\cdot 3}(-V^2)^3$$

$$(1-V^2)^{-\frac{1}{2}} = 1 + \frac{1}{2}V^2 + \frac{3}{8}V^4 + \frac{5}{16}V^6 + \cdots$$

$$2(1-\vec{V}\cdot\vec{v}) = 2 - 2(\vec{V}\cdot\vec{v})$$

$$\frac{2\left[1-(\vec{V}\cdot\vec{v})\right]}{\sqrt{1-V^2}} =$$

$$2$$
$$+V^2 - 2(\vec{V}\cdot\vec{v})$$
$$+\frac{3}{4}V^4 - V^2(\vec{V}\cdot\vec{v})$$
$$+\frac{5}{8}V^6 - \frac{3}{4}V^4(\vec{V}\cdot\vec{v})$$

$$\sqrt{1-V^2} = 1 - \frac{1}{2}V^2 - \frac{1}{8}V^4 - \frac{1}{16}V^6 + \cdots$$

$$(1-v^2) = 1 - v^2$$

$$1$$
$$-\frac{1}{2}V^2 - v^2$$
$$-\frac{1}{8}V^4 + \frac{1}{2}V^2 v^2$$
$$-\frac{1}{16}V^6 + \frac{1}{8}V^4 v^2$$

$$(1-v^2)\sqrt{1-V^2} = \left[1 + \left\{-\tfrac{1}{2}V^2 - v^2\right\} + \left\{-\tfrac{1}{8}V^4 + \tfrac{1}{2}V^2 v^2\right\} + \left\{-\tfrac{1}{16}V^6 + \tfrac{1}{8}V^4 v^2\right\} + \cdots\right]$$

$$\frac{1}{1-(\vec{V}\cdot\vec{v})} = 1 + (\vec{V}\cdot\vec{v}) + (\vec{V}\cdot\vec{v})^2 + (\vec{V}\cdot\vec{v})^3 + \cdots$$

$$-\frac{(1-v^2)\sqrt{1-V^2}}{\left\{1-(\vec{V}\cdot\vec{v})\right\}} =$$

$$-1$$

$$+\tfrac{1}{2}V^2 + v^2 - (\vec{V}\cdot\vec{v})$$

$$+\tfrac{1}{8}V^4 - \tfrac{1}{2}V^2 v^2 + \tfrac{1}{2}V^2(\vec{V}\cdot\vec{v}) + v^2(\vec{V}\cdot\vec{v}) - (\vec{V}\cdot\vec{v})^2$$

$$+\tfrac{1}{16}V^6 - \tfrac{1}{8}V^4 v^2 + \tfrac{1}{8}V^4(\vec{V}\cdot\vec{v}) - \tfrac{1}{2}V^2 v^2(\vec{V}\cdot\vec{v})$$

$$+\tfrac{1}{2}V^2(\vec{V}\cdot\vec{v})^2 + (\vec{V}\cdot\vec{v})^2 v^2 - (\vec{V}\cdot\vec{v})^3$$

$$
-\frac{(1-v^2)\sqrt{1-V^2}}{\{1-\vec{V}\cdot\vec{v}\}} = \left[\begin{aligned}
&-1\\
&+\left\{+\tfrac{1}{2}V^2+v^2-(\vec{V}\cdot\vec{v})\right\}\\
&+\left\{\begin{aligned}&+\tfrac{1}{8}V^4-\tfrac{1}{2}V^2v^2\\ &+\tfrac{1}{2}V^2(\vec{V}\cdot\vec{v})+v^2(\vec{V}\cdot\vec{v})\\ &-(\vec{V}\cdot\vec{v})^2\end{aligned}\right\}\\
&+\left\{\begin{aligned}&\tfrac{1}{16}V^6-\tfrac{1}{8}V^4v^2+\tfrac{1}{8}V^4(\vec{V}\cdot\vec{v})\\ &-\tfrac{1}{2}V^2v^2(\vec{V}\cdot\vec{v})\\ &+\tfrac{1}{2}V^2(\vec{V}\cdot\vec{v})^2+(\vec{V}\cdot\vec{v})^2v^2\\ &-(\vec{V}\cdot\vec{v})^3\end{aligned}\right\}
\end{aligned}\right.
$$

Comprobado

$$\left[\frac{2\{1-(\vec{V}\cdot\vec{v})\}}{\sqrt{1-V^2}} - \frac{(1-v^2)\sqrt{1-V^2}}{\{1-(\vec{V}\cdot\vec{v})\}}\right] =$$

$$1$$

$$+\left\{\frac{3}{2}V^2 + v^2 - 3\,\vec{V}\cdot\vec{v}\right\}$$

$$+\left\{\begin{array}{l}\frac{7}{8}V^4 - \frac{1}{2}V^2v^2 - \frac{1}{2}V^2(\vec{V}\cdot\vec{v}) \\ + v^2(\vec{V}\cdot\vec{v}) - (\vec{V}\cdot\vec{v})^2\end{array}\right\}$$

$$+\left\{\begin{array}{l}\frac{11}{16}V^6 - \frac{1}{8}V^4v^2 - \frac{5}{8}V^4(\vec{V}\cdot\vec{v}) \\ - \frac{1}{2}V^2v^2(\vec{V}\cdot\vec{v}) + \frac{1}{2}V^2(\vec{V}\cdot\vec{v})^2 \\ + (\vec{V}\cdot\vec{v})^2v^2 - (\vec{V}\cdot\vec{v})^3\end{array}\right\}$$

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$$+3 M^2 V^2 R^2 + 2 M^2 v^2 R^2 - 6 M^2 (\vec{V} \cdot \vec{v}) R^2$$

$$\cancel{-\frac{1}{4} M V^4 R} - 2 M^2 V^2 R^2 + \frac{1}{3} M^3 R^3$$

$$\mathcal{L} =$$

$$1$$

$$\frac{3}{2}V^2 + v^2 - 3(\vec{V}\cdot\vec{v}) + 2M\mathcal{R}$$

$$\frac{7}{8}V^4 - \frac{1}{2}V^2 v^2 - \frac{1}{2}V^2(\vec{V}\cdot\vec{v}) + v^2(\vec{V}\cdot\vec{v})$$

$$-(\vec{V}\cdot\vec{v})^2 + 3MV^2\mathcal{R} + 2Mv^2\mathcal{R} - 6M(\vec{V}\cdot\vec{v})\mathcal{R}$$

$$-1MV^2\mathcal{R} + 2M^2\mathcal{R}^2$$

dos

$$+\frac{11}{16}V^6 - \frac{1}{8}V^4 v^2 - \frac{5}{8}V^4(\vec{V}\cdot\vec{v})$$

$$-\frac{1}{2}V^2 v^2(\vec{V}\cdot\vec{v}) + \frac{1}{2}V^2(\vec{V}\cdot\vec{v})^2$$

$$+(\vec{V}\cdot\vec{v})^2 v^2 - (\vec{V}\cdot\vec{v})^3 + \cancel{\frac{7}{4}MV^4\mathcal{R}}$$

$$-1MV^2v^2\mathcal{R} - 1MV^2(\vec{V}\cdot\vec{v})\mathcal{R}$$

$$+2M(\vec{V}\cdot\vec{v})v^2\mathcal{R} - 2M(\vec{V}\cdot\vec{v})^2\mathcal{R}$$

$$\cancel{-\frac{3}{2}MV^4\mathcal{R}} - 1MV^2v^2\mathcal{R} + \frac{3}{2}MV^2(\vec{V}\cdot\vec{v})\mathcal{R}$$

$$\mathcal{L} =$$

$$1$$

$$\frac{3}{2}V^2 + v^2 - 3(\vec{V}\cdot\vec{v}) + 2M\mathcal{R}$$

$$+\frac{7}{8}V^4 - \frac{1}{2}V^2v^2 - \frac{1}{2}V^2(\vec{V}\cdot\vec{v}) + v^2(\vec{V}\cdot\vec{v})$$

$$-(\vec{V}\cdot\vec{v})^2 + 2MV^2\mathcal{R} + 2Mv^2\mathcal{R} - 6M(\vec{V}\cdot\vec{v})\mathcal{R}$$

$$+ 2M^2\mathcal{R}$$

$$+\frac{11}{16}V^6 - \frac{1}{8}V^4v^2 - \frac{5}{8}V^4(\vec{V}\cdot\vec{v})$$

$$-\frac{1}{2}V^2v^2(\vec{V}\cdot\vec{v}) + \frac{1}{2}V^2(\vec{V}\cdot\vec{v})^2$$

$$+(\vec{V}\cdot\vec{v})^2v^2 - (\vec{V}\cdot\vec{v})^3 - 2MV^2v^2\mathcal{R}$$

$$+2MV^2(\vec{V}\cdot\vec{v})\mathcal{R} + \underline{2M(\vec{V}\cdot\vec{v})v^2\mathcal{R}}$$

$$-2M(\vec{V}\cdot\vec{v})^2\mathcal{R} + M^2V^2\mathcal{R}^2$$

$$+2M^2v^2\mathcal{R}^2 - 6M^2(\vec{V}\cdot\vec{v})\mathcal{R}^2$$

$$+\frac{4}{3}M^3\mathcal{R}^3$$

$$\mathcal{L} \mathrel{\underset{\cdot}{=}}$$

$\underline{1}$

$$+\frac{3}{2}V^2 - 3(\vec{V}\cdot\vec{v}) + v^2 + 2M\mathcal{R}$$

$$+\frac{7}{8}V^4 - \frac{1}{2}V^2(\vec{V}\cdot\vec{v}) - (\vec{V}\cdot\vec{v})^2 - \frac{1}{2}V^2v^2 + (\vec{V}\cdot\vec{v})v^2$$
$$+3MV^2\mathcal{R} - 6M(\vec{V}\cdot\vec{v})\mathcal{R} + 2Mv^2\mathcal{R}$$
$$-MV^2\mathcal{R} + 2M^2\mathcal{R}^2$$

$$+\frac{11}{16}V^6 - \frac{5}{8}V^4(\vec{V}\cdot\vec{v}) + \frac{1}{2}V^2(\vec{V}\cdot\vec{v})^2 - (\vec{V}\cdot\vec{v})^3$$
$$-\frac{1}{8}V^4v^2 - \frac{1}{2}V^2(\vec{V}\cdot\vec{v})v^2 + (\vec{V}\cdot\vec{v})^2v^2$$
$$\cancel{+\frac{7}{4}MV^4\mathcal{R}} - MV^2(\vec{V}\cdot\vec{v})\mathcal{R} - 2M(\vec{V}\cdot\vec{v})^2\mathcal{R}$$
$$+2M(\vec{V}\cdot\vec{v})v^2\mathcal{R}$$
$$\cancel{-\frac{3}{2}MV^4\mathcal{R}} + 3MV^2(\vec{V}\cdot\vec{v})\mathcal{R} - MV^2v^2\mathcal{R}$$
$$+3M^2V^2\mathcal{R}^2 - 6M^2(\vec{V}\cdot\vec{v})\mathcal{R}^2 + 2M^2v^2\mathcal{R}^2$$
$$\cancel{-\frac{1}{4}MV^4\mathcal{R}} - 2M^2V^2\mathcal{R}^2 + \frac{4}{3}M^3\mathcal{R}^3$$

$$\mathcal{L} =$$

$$1$$

$$+ \tfrac{3}{2} V^2 - 3(\vec{V}\cdot\vec{v}) + v^2 + 2M\mathcal{R}$$

$$+ \tfrac{7}{8} V^4 - \tfrac{1}{2} V^2 (\vec{V}\cdot\vec{v}) - (\vec{V}\cdot\vec{v})^2 - \tfrac{1}{2} V^2 v^2 + (\vec{V}\cdot\vec{v}) v^2$$
$$+ 2MV^2\mathcal{R} - 6M(\vec{V}\cdot\vec{v})\mathcal{R} + 2Mv^2\mathcal{R}$$
$$+ 2M^2\mathcal{R}^2$$

$$+ \tfrac{11}{16} V^6 - \tfrac{5}{8} V^4 (\vec{V}\cdot\vec{v}) + \tfrac{1}{2} V^2 (\vec{V}\cdot\vec{v})^2 - (\vec{V}\cdot\vec{v})^3$$
$$- \tfrac{1}{8} V^4 v^2 - \tfrac{1}{2} V^2 (\vec{V}\cdot\vec{v}) v^2 + (\vec{V}\cdot\vec{v})^2 v^2$$
$$+ 2MV^2(\vec{V}\cdot\vec{v})\mathcal{R} - 2M(\vec{V}\cdot\vec{v})^2\mathcal{R}$$
$$+ 2M(\vec{V}\cdot\vec{v})v^2\mathcal{R} - MV^2v^2\mathcal{R}$$
$$+ M^2V^2\mathcal{R}^2 - 6M^2(\vec{V}\cdot\vec{v})\mathcal{R}^2$$
$$+ 2M^2v^2\mathcal{R}^2 + \tfrac{4}{3}M^3\mathcal{R}^3$$

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Desarrollo de $\vec{a}$ en función de $\vec{r}$, $\vec{V}$, $\vec{A}$, y $\vec{v}$. (hasta términos de 6º grado en c^{-1}.

Términos en $\vec{r}$

$$
\begin{array}{c|l}
\text{Grado en } c^{-1} & \\
\hline
2 & -M\mathcal{R}^3 \\
\hline
4 & -\frac{1}{2}MV^2\mathcal{R}^3 - Mv^2\mathcal{R}^3 + 4M(\vec{V}\cdot\vec{v})\mathcal{R}^3 \\
 & -M(\vec{A}\cdot\vec{r})\mathcal{R}^3 \\
\hline
5 & -3M(\vec{V}\cdot\vec{A})\mathcal{R}^2 + 4M(\vec{A}\cdot\vec{v})\mathcal{R}^2 \\
\hline
6 & +\frac{5}{8}MV^4\mathcal{R}^3 + \frac{3}{2}MV^2v^2\mathcal{R}^3 \\
 & -2MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})^2\mathcal{R}^3 \\
 & -\frac{3}{2}MV^2(\vec{A}\cdot\vec{r})\mathcal{R}^3 - M(\vec{A}\cdot\vec{r})v^2\mathcal{R}^3 \\
 & +4M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})\mathcal{R}^3
\end{array}
\;\Bigg]\,\vec{r}
$$

¡Comprobado!

grado en c^{-1}	Desarrollo de $\vec{a}$ en función de $\vec{r}$, $\vec{V}$, $\vec{A}$ y $\vec{v}$. (Hasta términos de 6. grado en c^{-1})
	Términos en $\vec{V}$
3	$-M\mathcal{R}^2 + 2Mr\mathcal{R}^3$
4	$-2M(\vec{r}\cdot\vec{v})\mathcal{R}^3$ correcto
5 correcto	$+\frac{1}{2}MV^2\mathcal{R}^2 + Mv^2\mathcal{R}^2$
	$-MV^2r\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})r\mathcal{R}^3$
	$+2M(\vec{A}\cdot\vec{r})r\mathcal{R}^3 - 2M(\vec{A}\cdot\vec{v})r\mathcal{R}^3$
6	$+MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$
	$+2M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3$
	$+2M(\vec{V}\cdot\vec{A})r\mathcal{R}^2$
	$-2M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3$

$\vec{V}$

en c^{-1}

Desarrollo de $\vec{a}$ en función de $\vec{r}$, $\vec{V}$, $\vec{A}$ y $\vec{v}$ (hasta términos de 6° grado en c^{-1}).

Términos en $\vec{A}$.

4	$2Mr\mathcal{R}^2$
5	$-2M(\vec{r}\cdot\vec{v})\mathcal{R}^2$
6	$+MV^2r\mathcal{R}^2 - 2M(\vec{V}\cdot\vec{v})r\mathcal{R}^2$

$$\left[\begin{array}{l} 2Mr\mathcal{R}^2 \\ -2M(\vec{r}\cdot\vec{v})\mathcal{R}^2 \\ +MV^2r\mathcal{R}^2 - 2M(\vec{V}\cdot\vec{v})r\mathcal{R}^2 \end{array}\right]\vec{A}$$

lo en c^{-1}

Desarrollo de $\vec{a}$ en función de $\vec{r}$, $\vec{V}$, $\vec{A}$ y $\vec{v}$. (Hasta términos de 6. grado en c^{-1}).

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Términos en $\vec{v}$

3
$$+M\mathcal{R}^2$$
$$-Mr\mathcal{R}^3$$

4
$$+2M(\vec{r}\cdot\vec{v})\mathcal{R}^3$$
$$-\tfrac{1}{2}MV^2\mathcal{R}^2-Mv^2\mathcal{R}^2$$

5
$$-M(\vec{A}\cdot\vec{r})r\mathcal{R}^3$$
$$+\tfrac{3}{2}MV^2r\mathcal{R}^3+Mv^2r\mathcal{R}^3$$
$$-2M(\vec{V}\cdot\vec{v})r\mathcal{R}^3-2M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

6
$$+M(\vec{V}\cdot\vec{A})r\mathcal{R}^2-2M(\vec{A}\cdot\vec{v})r\mathcal{R}^2$$
$$-MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3+2M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

$\vec{v}$

$(\vec{v}\cdot\vec{a})$ hasta términos de 6 grado en c^{-1}

Grado en c^{-1}

3 $-M(\vec{r}\cdot\vec{v})\mathcal{R}^3$

$-M(\vec{V}\cdot\vec{v})\mathcal{R}^2 + 2M(\vec{V}\cdot\vec{v})r\mathcal{R}^3$

4 $+Mv^2\mathcal{R}^2 - Mrv^2\mathcal{R}^3$

$-\frac{1}{2}MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 - M(\vec{r}\cdot\vec{v})v^2\mathcal{R}^3$

5 $+4M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3 - M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3$

$-2M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3 + 2M(\vec{A}\cdot\vec{v})r\mathcal{R}^2$

$+2M(\vec{r}\cdot\vec{v})v^2\mathcal{R}^3$

$-3M(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}^2 + 4M(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^2$

6 $+\frac{1}{2}MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^2 + M(\vec{V}\cdot\vec{v})v^2\mathcal{R}^2$

$-MV^2(\vec{V}\cdot\vec{v})r\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})^2r\mathcal{R}^3$

$+2M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})r\mathcal{R}^3 - \cancel{2M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{v})r}\mathcal{R}^3$

$-2M(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^2$

$-\frac{1}{2}MV^2v^2\mathcal{R}^2 - Mv^4\mathcal{R}^2$

$-M(\vec{A}\cdot\vec{r})rv^2\mathcal{R}^3 + \frac{3}{2}MV^2rv^2\mathcal{R}^3$

$+Mrv^4\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})rv^2\mathcal{R}^3$

$(\vec{v}\cdot\vec{a})$ hasta términos de 6. grado en c^{-1}

rado en c^{-1}

3 $-M(\vec{r}\cdot\vec{v})\mathcal{R}^3$

$-M(\vec{V}\cdot\vec{v})\mathcal{R}^2 + 2M(\vec{V}\cdot\vec{v})\, r\, \mathcal{R}^3$

4 $+Mv^2\mathcal{R}^2 - Mrv^2\mathcal{R}^3$

correcto → $-\frac{1}{2}MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 + M(\vec{r}\cdot\vec{v})v^2\mathcal{R}^3$

5 $+2M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3 + 2M(\vec{A}\cdot\vec{v})\, r\, \mathcal{R}^2$

$-M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3$

6 $-3M(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}^2 + 2M(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^2$

$+\frac{1}{2}MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^2 + M(\vec{V}\cdot\vec{v})v^2\mathcal{R}^2$

$-MV^2(\vec{V}\cdot\vec{v})\, r\, \mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})^2\, r\, \mathcal{R}^3$

$+2M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})\, r\, \mathcal{R}^3 \cancel{-2M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{v})\, r\, \mathcal{R}^3}$

$-\frac{1}{2}MV^2v^2\mathcal{R}^2 - Mv^4\mathcal{R}^2$

$-M(\vec{A}\cdot\vec{r})\, r v^2\mathcal{R}^3 + \frac{3}{2}MV^2 r v^2\mathcal{R}^3$

$+Mrv^4\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})\, r v^2\mathcal{R}^3$

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$(\vec{v}\cdot\vec{a})$ hasta términos de 6. grado en c^{-1}
(Comprobación pág. 212 cuaderno 101)

Grado en c^{-1}

3

$$-M(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

4

$$+Mv^2\mathcal{R}^2 - M(\vec{V}\cdot\vec{v})\mathcal{R}^2 - Mrv^2\mathcal{R}^3$$
$$+2M(\vec{V}\cdot\vec{v})r\mathcal{R}^3$$

5

$$+2M(\vec{A}\cdot\vec{v})r\mathcal{R}^2 - \frac{3}{2}MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

tres

$$-Mv^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 + MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 + 2M(\vec{r}\cdot\vec{v})v^2\mathcal{R}$$
$$+2M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3 + M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3$$
$$-2M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

6

$$-\frac{1}{2}MV^2v^2\mathcal{R}^2 - Mv^4\mathcal{R}^2$$
$$+\frac{1}{2}MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^2 + M(\vec{V}\cdot\vec{v})v^2\mathcal{R}^2$$
$$-M(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}^2 + 2M(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^2$$
$$-2M(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}^2 + \frac{3}{2}MV^2rv^2\mathcal{R}^3$$
$$+Mrv^4\mathcal{R}^3 - MV^2(\vec{V}\cdot\vec{v})r\mathcal{R}^3$$
$$-2M(\vec{V}\cdot\vec{v})v^2\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})^2r\mathcal{R}^3$$
$$-M(\vec{A}\cdot\vec{r})rv^2\mathcal{R}^3 + 2M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})r\mathcal{R}^3$$

Grado en c^{-1}

3

4

correcto →

5

$(\vec{v} \cdot \vec{a}) =$

$(\vec{v} \cdot \vec{a})$ hasta
en c^{-1}

$- M(\vec{r} \cdot \vec{v})$

$- M(\vec{V} \cdot \vec{v}) \mathcal{R}$

$+ M v^2 \mathcal{R}^2$

$- \frac{1}{2} M V^2 (\vec{r}$

$+ 2M(\vec{V} \cdot \vec{v})$

$- M(\vec{\mathcal{R}} \cdot \vec{r})$

$- 3M(\vec{V} \cdot \vec{\mathcal{R}}$

$+ \frac{1}{2} M V^2 ($

$- M V^2 (\vec{V} \cdot$

$+ 2M(\vec{V} \cdot$

$- M v^2 \mathcal{R}$

Grado en c^{-1}	$(\vec{v}\cdot\vec{a})$ hasta términos de 6. grado en c^{-1}
3	$-M(\vec{r}\cdot\vec{v})\mathcal{R}^3$
4	$-M(\vec{V}\cdot\vec{v})\mathcal{R}^2 + 2M(\vec{V}\cdot\vec{v})r\mathcal{R}^3$ $+Mv^2\mathcal{R}^2 - Mrv^2\mathcal{R}^3$
correcto → 5	$-\frac{1}{2}MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 + M(\vec{r}\cdot\vec{v})v^2\mathcal{R}^3$ $+2M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3 + 2M(\vec{A}\cdot\vec{v})r\mathcal{R}^2$ $-M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3$
correcto →	$-3M(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}^2 + 2M(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^2$ $+\frac{1}{2}MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^2 + M(\vec{V}\cdot\vec{v})v^2\mathcal{R}^2$ $-MV^2(\vec{V}\cdot\vec{v})r\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})^2 r\mathcal{R}^3$ $+2M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})r\mathcal{R}^3 - \frac{1}{2}MV^2v^2\mathcal{R}^2$ $-Mv^4\mathcal{R}^2 - M(\vec{A}\cdot\vec{r})rv^2\mathcal{R}^3$ $+\frac{3}{2}MV^2rv^2\mathcal{R}^3 + Mrv^4\mathcal{R}^3$ $-2M(\vec{V}\cdot\vec{v})v^2\mathcal{R}^3$

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Grado en c^{-1} | $(\vec{V}\cdot\vec{a})$ hasta términos de 6. grado en c^{-1}.

3 | $-M(\vec{V}\cdot\vec{r})\mathcal{R}^3$

4 | $-MV^2\mathcal{R}^2+2MV^2 r\,\mathcal{R}^3+M(\vec{V}\cdot\vec{v})\mathcal{R}^2$

$-M(\vec{V}\cdot\vec{v})r\,\mathcal{R}^3$

5 | $\cancel{-2MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3+2M(\vec{V}\cdot\vec{A})r\,\mathcal{R}^2}$

$\cancel{-2M(\vec{V}\cdot\vec{v})r\,\mathcal{R}^3}$

6 | $-3M(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{r})\mathcal{R}^2+4M(\vec{V}\cdot\vec{r})(\vec{A}\cdot\vec{v})\mathcal{R}^2$

correctos → $+\frac{1}{2}MV^4\mathcal{R}^2+MV^2v^2\mathcal{R}^2-MV^4r\,\mathcal{R}^3$

$-2MV^2(\vec{V}\cdot\vec{v})r\,\mathcal{R}^3+2MV^2(\vec{A}\cdot\vec{r})r\,\mathcal{R}^3$

$-2M(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}^2-\frac{1}{2}MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^2$

$-M(\vec{V}\cdot\vec{v})v^2\mathcal{R}^2-M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})r\,\mathcal{R}^3$

$+\frac{3}{2}MV^2(\vec{V}\cdot\vec{v})r\,\mathcal{R}^3+M(\vec{V}\cdot\vec{v})v^2r\,\mathcal{R}^3$

$-2M(\vec{V}\cdot\vec{v})^2r\,\mathcal{R}^3$

#

Grado en c^{-4}

3

4

5

$$(\vec{V}\cdot\vec{a}) =$$

6

$(\vec{V}\cdot\vec{a})$ hasta

$-M(\vec{V}\cdot\vec{r})$

$-MV^2 r^2$

$-M(\vec{V}\cdot\vec{v})$

$-\frac{1}{2}MV^2(\vec{V}$

$+4M(\vec{V}\cdot\vec{r})$

$-2MV^2(\vec{r}\cdot$

$+2M(\vec{V}\cdot\vec{v})$

$-3M(\vec{V}\cdot\vec{A}$

$+\frac{1}{2}MV^4$

$-\frac{1}{2}MV^2$

$-2M(\vec{V}\cdot\vec{A}$

$-M(\vec{V}\cdot\vec{v})$

Grado en c^{-1} — $(\vec{V}\cdot\vec{a})$ hasta términos de 6. grado en c^{-1}

3

$$-M(\vec{V}\cdot\vec{r})\mathcal{R}^3$$

4

$$-MV^2\mathcal{R}^2+2MV^2r\mathcal{R}^3+M(\vec{V}\cdot\vec{v})\mathcal{R}^2$$
$$-M(\vec{V}\cdot\vec{v})r\mathcal{R}^3$$

5

$$-\tfrac{1}{2}MV^2(\vec{V}\cdot\vec{r})\mathcal{R}^3-M(\vec{V}\cdot\vec{r})v^2\mathcal{R}^3$$
$$+4M(\vec{V}\cdot\vec{r})(\vec{V}\cdot\vec{v})\mathcal{R}^3-M(\vec{V}\cdot\vec{r})(\vec{A}\cdot\vec{r})\mathcal{R}^3$$
$$-2MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3+2M(\vec{V}\cdot\vec{A})r\mathcal{R}^2$$
$$+2M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

6

$$-3M(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{r})\mathcal{R}^2+4M(\vec{V}\cdot\vec{r})(\vec{A}\cdot\vec{v})\mathcal{R}^2$$
$$+\tfrac{1}{2}MV^4\mathcal{R}^2+MV^2v^2\mathcal{R}^2-MV^4r\mathcal{R}^3$$
$$-\tfrac{1}{2}MV^2(\vec{V}\cdot\vec{v})r\mathcal{R}^3+2MV^2(\vec{A}\cdot\vec{r})r\mathcal{R}^3$$
$$-2M(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}^2-\tfrac{1}{2}MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^2$$
$$-M(\vec{V}\cdot\vec{v})v^2\mathcal{R}^2-M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})r\mathcal{R}^3$$
$$+M(\vec{V}\cdot\vec{v})v^2r\mathcal{R}^3-2M(\vec{V}\cdot\vec{v})^2r\mathcal{R}^3$$

¡Comprobado!

Grado en c^{-1}

$$\frac{\partial \mathcal{L}}{\partial \vec{v}} =$$

1 $\quad -3\vec{V} + 2\vec{v}$

3 $\quad -\frac{1}{2}V^2\vec{V} - 2(\vec{V}\cdot\vec{v})\vec{V} - V^2\vec{v} + v^2\vec{V} + 2(\vec{V}\cdot\vec{v})\vec{v}$

$\quad - 6M\mathcal{R}\vec{V} + 4M\mathcal{R}\vec{v}$

5 $\quad -\frac{5}{8}V^4\vec{V} + V^2(\vec{V}\cdot\vec{v})\vec{V} - 3(\vec{V}\cdot\vec{v})^2\vec{V}$

$\quad -\frac{1}{4}V^4\vec{v} - \frac{1}{2}V^2v^2\vec{V} - V^2(\vec{V}\cdot\vec{v})\vec{v}$

$\quad + 2(\vec{V}\cdot\vec{v})v^2\vec{V} + 2(\vec{V}\cdot\vec{v})^2\vec{v} + 2MV^2\mathcal{R}\vec{V}$

$\quad - 4M(\vec{V}\cdot\vec{v})\mathcal{R}\vec{V} + 2Mv^2\mathcal{R}\vec{V}$

$\quad + 4M(\vec{V}\cdot\vec{v})\mathcal{R}\vec{v} - 2MV^2\mathcal{R}\vec{v}$

$\quad - 6M^2\mathcal{R}^2\vec{V} + 4M^2\mathcal{R}^2\vec{v}.$

Comprobado

$$\frac{\partial \vec{V}}{\partial t} = \frac{r}{r - \vec{r}\cdot\vec{V}}\vec{A} = r\mathcal{R}\vec{A}$$

$$(\vec{v}\cdot\nabla)\vec{V} = -\frac{(\vec{r}\cdot\vec{v})}{(r - \vec{r}\cdot\vec{V})}\vec{A} = -(\vec{r}\cdot\vec{v})\mathcal{R}\vec{A}$$

$$\boxed{\frac{d\vec{V}}{dt} = r\mathcal{R}\vec{A} - (\vec{r}\cdot\vec{v})\dot{\mathcal{R}}\vec{A}} \longleftarrow$$

$$\boxed{-3\frac{d\vec{V}}{dt} = \underset{2}{-3r\mathcal{R}\vec{A}} + \underset{3}{3(\vec{r}\cdot\vec{v})\mathcal{R}\vec{A}}}$$

$$\begin{array}{llc} & & \text{Grado en } c^{-1} \\ -3\dfrac{d\vec{V}}{dt} = & \left[\begin{array}{l} -3r\mathcal{R} \\ \hline \\ +3(\vec{r}\cdot\vec{v})\mathcal{R} \\ \hline \end{array}\right]\vec{A} & \begin{array}{c} 2 \\ \\ \\ 3 \end{array} \end{array}$$

Grado en c^{-1}	$\vec{r}$	$\vec{V}$
2	$-2M\mathcal{R}^3$	
3		$-2M\mathcal{R}^2$ $+4Mr\mathcal{R}^3$
4	$-4MV^2\mathcal{R}^3$ $-2Mv^2\mathcal{R}^3$ $+8M(\vec{V}\cdot\vec{v})\mathcal{R}^3$ $-2M(\vec{A}\cdot\vec{r})\mathcal{R}^3$	$-4M(\vec{r}\cdot\vec{v})\mathcal{R}^3$
5	$-6M(\vec{V}\cdot\vec{A})\mathcal{R}^2$ $+8M(\vec{A}\cdot\vec{v})\mathcal{R}^2$	$+MV^2\mathcal{R}^2$ $+2Mv^2\mathcal{R}^2$ $-2MV^2r\mathcal{R}^3$ $-4M(\vec{V}\cdot\vec{v})r\mathcal{R}^3$ $+4M(\vec{A}\cdot\vec{r})r\mathcal{R}^3$
6	Tres $+\frac{5}{4}MV^4\mathcal{R}^3$ $+3MV^2v^2\mathcal{R}^3$ $-4MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^3$ $-4M(\vec{V}\cdot\vec{v})^2\mathcal{R}^3$ $-3MV^2(\vec{A}\cdot\vec{r})\mathcal{R}^3$	$-4M(\vec{A}\cdot\vec{v})r\mathcal{R}^3$ $+2MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$ $+4M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3$ $+4M(\vec{V}\cdot\vec{A})r\mathcal{R}^2$ $-4M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3$

$\boxed{2\vec{a} =}$

$+4M$

$-4M$

$+2M$

$-4M($

$\boxed{2\vec{a} =}$

$[\ \ ]\ \vec{A}$	$[\ \ ]\ \vec{v}$
	$+2M\mathcal{R}^2$ $-2Mr\mathcal{R}^3$
$+4Mr\mathcal{R}^2$	$+4M(\vec{r}\cdot\vec{v})\mathcal{R}^3$
$-4M(\vec{r}\cdot\vec{v})\mathcal{R}^2$	$-MV^2\mathcal{R}^2$ $-2Mv^2\mathcal{R}^2$ $-2M(\vec{A}\cdot\vec{r})r\mathcal{R}^3$ $+3MV^2r\mathcal{R}^3$ $+2Mv^2r\mathcal{R}^3$ $-4M(\vec{V}\cdot\vec{v})r\mathcal{R}^3$
$+2MV^2r\mathcal{R}^2$ $-4M(\vec{V}\cdot\vec{v})r\mathcal{R}^2$	$-4M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3$ $+2M(\vec{V}\cdot\vec{A})r\mathcal{R}^2$ $-4M(\vec{A}\cdot\vec{v})r\mathcal{R}^2$ $-2MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$ $+4M(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3$

$$\vec{V}\cdot\frac{d\vec{V}}{dt} = r\,\mathcal{R}\,(\vec{V}\cdot\vec{A}) - (\vec{r}\cdot\vec{v})\,\mathcal{R}\,(\vec{V}\cdot\vec{A})$$

$$-\left(\vec{V}\cdot\frac{d\vec{V}}{dt}\right)\vec{V} = -\underset{4}{(\vec{V}\cdot\vec{A})_r}\,\mathcal{R}\,\vec{V} + (\vec{V}\cdot\vec{A})\underset{5}{(\vec{r}\cdot\vec{v})}\,\mathcal{R}\,\vec{V}$$

$$-\frac{1}{2}V^2\frac{d\vec{V}}{dt} = -\frac{1}{2}\underset{4}{V^2_r}\,\mathcal{R}\,\vec{A} + \frac{1}{2}V^2\underset{5}{(\vec{r}\cdot\vec{v})}\,\mathcal{R}\,\vec{A}$$

$$\frac{d}{dt}\left\{-\frac{1}{2}V^2\vec{V}\right\} =$$

$$\begin{array}{c} \text{Gradoen} \\ \tau^{-1} \\ 4 \\ \\ 5 \end{array}
\quad
\begin{bmatrix} -(\vec{V}\cdot\vec{A})\,r\,\mathcal{R} \\ \\ +(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\,\mathcal{R} \\ \\ \end{bmatrix}\vec{V}
+
\begin{bmatrix} -\frac{1}{2}V^2_r\,\mathcal{R} \\ \\ +\frac{1}{2}V^2(\vec{r}\cdot\vec{v})\,\mathcal{R} \\ \\ \end{bmatrix}\vec{A}$$

Grado en c^{-1}

$$
\begin{array}{ll}
4 & +\left[\begin{array}{l}
+2M(\vec{V}\cdot\vec{r})\mathcal{R}^3 \\
-2(\vec{A}\cdot\vec{v})\, r\, \mathcal{R} \\
+2M V^2 \mathcal{R}^2 \\
-4M V^2 r\, \mathcal{R}^3 \\
-2M(\vec{V}\cdot\vec{v})\mathcal{R}^2 \\
+2M(\vec{V}\cdot\vec{v})\, r\, \mathcal{R}^3 \\
+2(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})\,\mathcal{R} \\
+M V^2(\vec{V}\cdot\vec{r})\mathcal{R}^3 \\
+2M(\vec{V}\cdot\vec{r})\, v^2\, \mathcal{R}^3 \\
-8M(\vec{V}\cdot\vec{r})(\vec{V}\cdot\vec{v})\mathcal{R}^3 \\
+2M(\vec{V}\cdot\vec{r})(\vec{A}\cdot\vec{r})\mathcal{R}^3 \\
+4M V^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 \\
-4M(\vec{V}\cdot\vec{A})\, r\, \mathcal{R}^2 \\
-4M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3
\end{array}\right]\vec{V} \\
5 & \\
6 &
\end{array}
$$

$$\frac{d}{dt}\left\{-2(\vec{V}\cdot\vec{v})\vec{V}\right\} =$$

$$+\begin{bmatrix} -2(\vec{V}\cdot\vec{v})\, r\, \mathcal{H} \\ +2(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\, \mathcal{H} \end{bmatrix}\vec{A}$$

$\frac{d}{dt}\{-\bar{V}$

Grado en c^{-1}	$+[\ \ ]\vec{r}\,+$	$[\ \ ]\vec{V}\,+$	$[\ \ ]$
4	$+MV^2\mathcal{R}^3$		
5		$+MV^2\mathcal{R}^2$ $-2MV^2 r\mathcal{R}^3$	
	$+\frac{1}{2}MV^4\mathcal{R}^3$ $+MV^2 v^2\mathcal{R}^3$ $-4MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^3$ $+MV^2(\vec{A}\cdot\vec{r})\mathcal{R}^3$	$+2MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$	$-2MV^2 r$

$$\frac{d}{dt}\left\{-V^2\vec{v}\right\} =$$

$$-MV^2\mathcal{R}^2$$
$$2MV^2 r\mathcal{R}^3$$
$$2MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

$$\Big]\vec{V} + \Big[\; -2MV^2 r\mathcal{R}^2 \;\Big]\vec{A} + \Big[\; -2(\vec{V}\cdot\vec{A})r\mathcal{R} - MV^2\mathcal{R}^2 + MV^2 r\mathcal{R}^3 + 2(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R} \mp 2MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 \;\Big]\vec{V}$$

Grado en c^{-1}

$$+\frac{d}{dt}\left(v^2\vec{V}\right)=$$

$$\left[\begin{array}{l}-2M(\vec{r}\cdot\vec{v})\mathcal{R}^3\\ -2M(\vec{V}\cdot\vec{v})\mathcal{R}^2\\ +4M(\vec{V}\cdot\vec{v})r\mathcal{R}^3\\ +2Mv^2\mathcal{R}^2\\ -2Mrv^2\mathcal{R}^3\\ -MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3\\ +2M(\vec{r}\cdot\vec{v})v^2\mathcal{R}^3\\ +4M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3\\ +4M(\vec{A}\cdot\vec{v})r\mathcal{R}^2\\ -2M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3\end{array}\right]\vec{V}+\left[\begin{array}{l}+rv^2\mathcal{R}\\ -(\vec{r}\cdot\vec{v})v^2\mathcal{R}\end{array}\right]\vec{A}$$

4

5

$$\frac{d}{dt}\left[2(\vec{V}\cdot\vec{v})\,\vec{v}\right] =$$

$$\begin{bmatrix} -2M(\vec{V}\cdot\vec{v})\mathcal{R}^3 \\ -MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^3 \\ -2M(\vec{V}\cdot\vec{v})v^2\mathcal{R}^3 \\ +8M(\vec{V}\cdot\vec{v})^2\mathcal{R}^3 \\ -2M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})\mathcal{R}^3 \end{bmatrix}\vec{r} + \begin{bmatrix} -2M(\vec{V}\cdot\vec{v})\mathcal{R}^2 \\ +4M(\vec{V}\cdot\vec{v})r\,\mathcal{R}^3 \\ -4M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3 \end{bmatrix}\vec{V} + \begin{bmatrix} 2M(\vec{V}\cdot\vec{v})r\,\mathcal{R}^2 \end{bmatrix}\vec{A}$$

Grado en c^{-1}

$\vec{v}$

$$
\begin{aligned}
&\Big[-2M(\vec{V}\cdot\vec{r})\mathcal{R}^3 && 4\\
&+2(\vec{A}\cdot\vec{v})r\mathcal{R} && \\
&-2MV^2\mathcal{R}^2 && 5\\
&+4MV^2r\mathcal{R}^3 && \\
&+2M(\vec{V}\cdot\vec{v})\mathcal{R}^2 && \\
&-2M(\vec{V}\cdot\vec{v})r\mathcal{R}^3 && \\
&+2M(\vec{V}\cdot\vec{v})\mathcal{R}^2 && \\
&-2M(\vec{V}\cdot\vec{v})r\mathcal{R}^3 && \\
&-2(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R} && \\
&-MV^2(\vec{V}\cdot\vec{r})\mathcal{R}^3 && 6\\
&-2M(\vec{V}\cdot\vec{r})v^2\mathcal{R}^3 && \\
&+8M(\vec{V}\cdot\vec{r})(\vec{V}\cdot\vec{v})\mathcal{R}^3 && \\
&-2M(\vec{V}\cdot\vec{r})(\vec{A}\cdot\vec{r})\mathcal{R}^3 && \\
&-4MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 && \\
&+4M(\vec{V}\cdot\vec{A})r\mathcal{R}^3 && \\
&+4M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3 && \\
&+4M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3 \Big]
\end{aligned}
$$

Grado en c^{-1}

$$-6M\frac{d}{dt}\left(\mathcal{R}\vec{V}\right) =$$

$$\begin{bmatrix} +6M(\vec{r}\cdot\vec{v})\mathcal{R}^3 \\ -6M(\vec{V}\cdot\vec{r})\mathcal{R}^3 \\ -6M(\vec{V}\cdot\vec{v})\mathcal{R}^2 \\ -6M(\vec{A}\cdot\vec{r})r\mathcal{R}^3 \\ +6MV^2r\mathcal{R}^3 \\ -6MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 \\ +6M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3 \end{bmatrix}\vec{V} + \begin{bmatrix} -6Mr\mathcal{R}^2 \\ +6M(\vec{r}\cdot\vec{v})\mathcal{R}^2 \\ \\ \\ \\ \\ \end{bmatrix}\vec{A}$$

4

5

6

1

Grado en c^{-1}	$\left[\ \right]\vec{r}\ +$	$\left[\ \right]\vec{V}\ +$	$\frac{d}{dt}\left\{ 4M\mathcal{R}\vec{v}\right.$
4	$-4M^2\mathcal{R}^4$		
5		$-4M^2\mathcal{R}^2$ $+8M^2 r\mathcal{R}^3$	
6	$-2M^2V^2\mathcal{R}^3$ $-4M^2v^2\mathcal{R}^3$ $+16M^2(\vec{V}\cdot\vec{v})\mathcal{R}^3$ $-4M^2(\vec{A}\cdot\vec{r})\mathcal{R}^3$	$-8M^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$	$8M^2 r\mathcal{R}^2$

$$\frac{d}{dt}\left\{ 4M\mathcal{R}\vec{v} \right\}$$

$\mathcal{R}^2$

$\mathcal{R}^3$

$\vec{v})\mathcal{R}^3$

$$\Big]\vec{V} + \Big[8M^2 r\mathcal{R}^2 \Big]\vec{A} + \Big[-4M(\vec{r}\cdot\vec{v})\mathcal{R}^3 + 4M(\vec{V}\cdot\vec{r})\mathcal{R}^3 + 4M^2\mathcal{R}^2 - 4M^2 r\mathcal{R}^3 + 4M(\vec{V}\cdot\vec{v})\mathcal{R}^2 + 4M(\vec{A}\cdot\vec{r})r\mathcal{R}^3 - 4MV^2 r\mathcal{R}^3 + 8M^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 + 4MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 - 4M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3 \Big]\vec{v}$$

$$\nabla\left(-\frac{3}{2}V^2\right)=$$

$$\boxed{\nabla\left(-\frac{3}{2}V^2\right)=+3(\vec{V}\cdot\vec{A})\,\mathcal{R}\,\vec{r}}$$ 3

$$+\nabla(3\,\vec{V}\cdot\vec{v})=$$

$$-3(\vec{A}\cdot\vec{v})\,\mathcal{R}\,\vec{r}$$

$$\boxed{\nabla(3\,\vec{V}\cdot\vec{v})=-3(\vec{A}\cdot\vec{v})\,\mathcal{R}\,\vec{r}}$$ 3

$$\nabla(-2M\mathcal{R})=$$

$$\nabla(-2M\mathcal{R})=\left[+\underset{2}{2M\mathcal{R}^3\vec{r}}-\underset{4}{2MV^2\mathcal{R}^3\vec{r}}+\underset{4}{2M(\vec{A}\cdot\vec{r})\mathcal{R}^3\vec{r}}-\underset{3}{2M\mathcal{R}^2\vec{V}}\right]$$

Grado en $\vec{c}$	$\nabla(-2M\mathcal{R})=[\ \ ]\,\vec{r}$	$+[\ \ ]\,\vec{V}$
2	$2M\mathcal{R}^3$	
3		$-2M\mathcal{R}^2$
4	$-2MV^2\mathcal{R}^3$ $+2M(\vec{A}\cdot\vec{r})\mathcal{R}^3$	

$$\boxed{\nabla\left(-\frac{7}{8}V^4\right) = +\frac{7}{2}V^2(\vec{V}\cdot\vec{A})\mathcal{X}\vec{r}}$$ 5

$$\nabla\left[+\frac{1}{2}V^2(\vec{V}\cdot\vec{v})\right] =$$

$$-\frac{1}{2}V^2(\vec{A}\cdot\vec{v})\mathcal{X}\vec{r} - (\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{v})\mathcal{X}\vec{r}$$

4 5

Grado en c^{-1}

$$\boxed{\begin{aligned}&\nabla\left[\frac{1}{2}V^2(\vec{V}\cdot\vec{v})\right] = \\ &-\frac{1}{2}V^2(\vec{A}\cdot\vec{v})\mathcal{X}\vec{r} \\ &-(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{v})\mathcal{X}\vec{r}\end{aligned}}$$

4

5

$$\nabla(\vec{V}\cdot\vec{v})^2 =$$

$$-2(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{v})\mathcal{X}\vec{r}$$

$$\boxed{\nabla(\vec{V}\cdot\vec{v})^2 = -2(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{v})\mathcal{X}\vec{r}}$$

5

$$\nabla\left[\tfrac{1}{2} V^2 v^2\right] =$$

$$- (\vec{V} \cdot \vec{A})\, v^2 \mathcal{R}\, \vec{r}$$

$$\boxed{\nabla\left[\tfrac{1}{2} V^2 v^2\right] = - (\vec{V} \cdot \vec{A})\, v^2 \mathcal{R}\, \vec{r}}$$

5

$$\nabla\left[-(\vec{V} \cdot \vec{v})\, v^2\right] =$$

$$+ (\vec{A} \cdot \vec{v})\, v^2 \mathcal{R}\, \vec{r}$$

$$\boxed{\nabla\left[-(\vec{V} \cdot \vec{v})\, v^2\right] = + (\vec{A} \cdot \vec{v})\, v^2 \mathcal{R}\, \vec{r}}$$

5

$$\nabla\left[-2M V^2 \mathcal{R}\right] =$$

$$+ 4M (\vec{V} \cdot \vec{A})\, \mathcal{R}^2 \vec{r} + 2M V^2 \mathcal{R}^3 \vec{r}$$

5 4

6

$$+ 2M V^4 \mathcal{R}^3 \vec{r} + 2M V^2 (\vec{A} \cdot \vec{r})\, \mathcal{R}^3 \vec{r}$$

6

$$- 2M V^2 \mathcal{R}^2 \vec{V}$$

5

Grado en c^{-1}

$$\nabla\left[-2MV^2\mathcal{R}\right] =$$

4

$$\left[+2MV^2\mathcal{R}^3 + 4M(\vec{V}\cdot\vec{A})\mathcal{R}^2\right.$$

5

$$\left.-2MV^4\mathcal{R}^3 + 2MV^2(\vec{A}\cdot\vec{r})\mathcal{R}^3\right]\vec{r} + \left[-2MV^2\mathcal{R}^2\right]\vec{V}$$

6

$$\nabla\left[+6M(\vec{V}\cdot\vec{v})\mathcal{R}\right] =$$

$$-6M\overset{5}{(\vec{A}\cdot\vec{v})}\mathcal{R}^2\vec{r} - 6M\overset{4}{(\vec{V}\cdot\vec{v})}\mathcal{R}^3\vec{r}$$
$$+6MV^2\overset{6}{(\vec{V}\cdot\vec{v})}\mathcal{R}^3\vec{r} - 6M(\vec{V}\cdot\vec{v})\overset{6}{(\vec{A}\cdot\vec{r})}\mathcal{R}^3\vec{r}$$
$$+6M\underset{5}{(\vec{V}\cdot\vec{v})}\mathcal{R}^2\vec{V}$$

$$\nabla\left[6M(\vec{V}\cdot\vec{v})\mathcal{R}\right] =$$

Grado en c^{-1}	$\vec{r}$	$\vec{V}$
4	$-6M(\vec{V}\cdot\vec{v})\mathcal{R}^3$	
5	$-6M(\vec{A}\cdot\vec{v})\mathcal{R}^2$	$+6M(\vec{V}\cdot\vec{v})\mathcal{R}^2$
6	$+6M(V^2)(\vec{V}\cdot\vec{v})\mathcal{R}^3$ $-6M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})\mathcal{R}^3$	

$$\left[\ \cdots\ \right]\vec{r} + \left[\ \cdots\ \right]\vec{V}$$

$$\nabla\left[-2M v^2\mathcal{R}\right]$$

Grado en c^{-1}	$\vec{r}$	$\vec{V}$
4	$+2M v^2\mathcal{R}^3$	$-2M v^2\mathcal{R}^3$ (5)
6	$-2M V^2 v^2\mathcal{R}^3$ $+2M(\vec{A}\cdot\vec{r})v^2\mathcal{R}^3$	

$$\left[\ \cdots\ \right]\vec{r} + \left[\ \cdots\ \right]\vec{V}$$

$$\nabla\left[-2M v^2\mathcal{R}\right]$$

Grado en c^{-1}	$\vec{r}$	$\vec{V}$
4	$+2M v^2\mathcal{R}^3$	
5		$-2M v^2\mathcal{R}^3$
6	$-2M V^2 v^2\mathcal{R}^3$ $+2M(\vec{A}\cdot\vec{r})v^2\mathcal{R}^3$	

$$\left[\ \cdots\ \right]\vec{r} + \left[\ \cdots\ \right]\vec{V}$$

Grado en c^{-1}

$$\nabla\left[2M^2\mathcal{R}^2\right]$$

Grado en c^{-1}			
4	$\left[+4M^2\mathcal{R}^4\right.$	$\vec{r}\ +$	$\left[-4M^2\mathcal{R}^3\right]\vec{V}$
5			
6	$-4M^2V^2\mathcal{R}^4$ $\left.+4M^2(\vec{A}\cdot\vec{r})\mathcal{R}^4\right]$		

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \vec{v}}\right) - \nabla \mathcal{L} \quad \text{hasta}$$

Orden en c^{-1}	$\vec{r}$	$\vec{V}$
2	$-2M\mathcal{R}^3$ $+2M\mathcal{R}^3$	
3	$+3(\vec{V}\cdot\vec{A})\mathcal{R}$ $-3(\vec{A}\cdot\vec{v})\mathcal{R}$	$-2M\mathcal{R}^2$ $+4Mr\mathcal{R}^3$ $-2M\mathcal{R}^2$
4	$\cancel{+MV^2\mathcal{R}^3}$ $\cancel{-2Mv^2\mathcal{R}^3}$ $\cancel{+8M(\vec{V}\cdot\vec{v})\mathcal{R}^3}$ $-2M(\vec{A}\cdot\vec{r})\mathcal{R}^3$ $\cancel{+MV^2\mathcal{R}^3}$ $\cancel{-2M(\vec{V}\cdot\vec{v})\mathcal{R}^3}$ $\cancel{-4M^2\mathcal{R}^4}$ $\cancel{-2MV^2\mathcal{R}^3}$ $+2M(\vec{A}\cdot\vec{r})\mathcal{R}^3$ $-\frac{1}{2}V^2(\vec{A}\cdot\vec{v})\mathcal{R}$ $\cancel{+2MV^2\mathcal{R}^3}$ $\cancel{-6M(\vec{V}\cdot\vec{v})\mathcal{R}^3}$ $\cancel{+2Mv^2\mathcal{R}^3}$ $\cancel{+4M^2\mathcal{R}^4}$	$\cancel{-4M(\vec{r}\cdot\vec{v})\mathcal{R}^3}$ $-(\vec{V}\cdot\vec{A})r\mathcal{R}$ $+2M(\vec{V}\cdot\vec{r})\mathcal{R}^3$ $-2(\vec{A}\cdot\vec{v})r\mathcal{R}$ $\cancel{-2M(\vec{r}\cdot\vec{v})\mathcal{R}^3}$ $\cancel{+6M(\vec{r}\cdot\vec{v})\mathcal{R}^3}$ $-6M(\vec{V}\cdot\vec{r})\mathcal{R}^3$ $\cancel{2M\mathcal{R}^2}$

hasta términos de 4. grado en c^{-1}

$\vec{A}$	$\vec{v}$
$-3r\mathcal{R}$	
$+3(\vec{r}\cdot\vec{v})\mathcal{R}$	$+2M\mathcal{R}^2$ $-2Mr\mathcal{R}^3$
$+4Mr\mathcal{R}^2$ $-\frac{1}{2}V^2 r\mathcal{R}$ $-2(\vec{V}\cdot\vec{v})r\mathcal{R}$ $+rv^2\mathcal{R}$ $-6Mr\mathcal{R}^2$	$+\cancel{4M(\vec{r}\cdot\vec{v})\mathcal{R}^3}$ $-2(\vec{V}\cdot\vec{A})r\mathcal{R}$ $-2M(\vec{V}\cdot\vec{r})\mathcal{R}^3$ $+2(\vec{A}\cdot\vec{v})r\mathcal{R}$ $-\cancel{4M(\vec{r}\cdot\vec{v})\mathcal{R}^3}$ $+4M(\vec{V}\cdot\vec{r})\mathcal{R}^3$

Grado en c^{-1}	$\vec{F}$	$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \vec{V}}\right) - \nabla \mathcal{L}$ hasta $\vec{V}$
2		
3	$+3(\vec{V}\cdot\vec{A})\mathcal{R}$ $-3(\vec{A}\cdot\vec{v})\mathcal{R}$	
4	$-\frac{1}{2}V^2(\vec{A}\cdot\vec{v})\mathcal{R}$	$-(\vec{V}\cdot\vec{A})\,r\,\mathcal{R}$ $-2(\vec{A}\cdot\vec{V})\,r\,\mathcal{R}$

hasta términos de 4. grado en c^{-1}

$\vec{A}$	$\vec{V}$
$-3r\mathcal{R}$	
$+3(\vec{r}\cdot\vec{v})\mathcal{R}$	
$-2Mr\mathcal{R}^2$	$-2(\vec{V}\cdot\vec{A})r\mathcal{R}$
$-\frac{1}{2}V^2 r\mathcal{R}$	$+2(\vec{A}\cdot\vec{v})r\mathcal{R}$
$-2(\vec{V}\cdot\vec{v})r\mathcal{R}$	
$+rv^2\mathcal{R}$	

$$\mathcal{R} = \left\{ r - (\vec{V} \cdot \vec{r}) \right\}^{-1} = r^{-1} \left\{ 1 - \frac{\vec{V} \cdot \vec{r}}{r} \right\}^{-1}$$

$$\mathcal{R} = r^{-1} \left[1 + \frac{(\vec{V} \cdot \vec{r})}{r} + \frac{(\vec{V} \cdot \vec{r})^2}{r^2} + \frac{(\vec{V} \cdot \vec{r})^3}{r^3} + \frac{(\vec{V} \cdot \vec{r})^4}{r^4} + \ldots \right]$$

$$\boxed{\mathcal{R} = \left\{ \frac{1}{r} + \frac{(\vec{V} \cdot \vec{r})}{r^2} + \frac{(\vec{V} \cdot \vec{r})^2}{r^3} + \frac{(\vec{V} \cdot \vec{r})^3}{r^4} + \frac{(\vec{V} \cdot \vec{r})^4}{r^5} + \ldots \right]}$$

$$\boxed{2M\mathcal{R} = \left[\frac{2M}{r} + \frac{2M(\vec{V} \cdot \vec{r})}{r^2} + \frac{2M(\vec{V} \cdot \vec{r})^2}{r^3} + \frac{2M(\vec{V} \cdot \vec{r})^3}{r^4} + \frac{2M(\vec{V} \cdot \vec{r})^4}{r^5} + \ldots \right.}$$

Gradien c^{-1}

$$\mathcal{L} =$$

0	2	3	4
1	$+\frac{3}{2}V^2$	$+\frac{2M(\vec{V}\cdot\vec{r})}{r^2}$	
	$-3\vec{V}\cdot\vec{v}$		
	$+v^2$		
	$+2Mr$		

$\mathcal{L}$ en función de r, V, v, $(\vec{V}\cdot\vec{r})$, $(\vec{V}\cdot\vec{v})$

Grado en ε^{-1} →	0	1	2	3	4
→	1		$+\frac{3}{2}V^2$	$+\frac{2M(\vec{V}\cdot\vec{r})}{r^2}$	$+\frac{7}{8}V^4$
			$-3(\vec{V}\cdot\vec{v})$		$-\frac{1}{2}V^2(\vec{V}\cdot\vec{v})$
			$+v^2$		$-(\vec{V}\cdot\vec{v})^2$
			$+\frac{2M}{r}$		$-\frac{1}{2}V^2v^2$
					$+(\vec{V}\cdot\vec{v})v^2$
					$+\frac{2M(\vec{V}\cdot\vec{r})^2}{r^3}$
					$+\frac{2MV^2}{r}$
					$-\frac{6M(\vec{V}\cdot\vec{v})}{r}$
					$+\frac{2Mv^2}{r}$
					$+\frac{2M^2}{r^2}$

$\mathcal{L} =$

$$+ \frac{2M(\vec{V}\cdot\vec{r})^3}{r^4}$$

$$+ \frac{2M V^2(\vec{V}\cdot\vec{r})}{r^2}$$

$$- \frac{6M(\vec{V}\cdot\vec{r})(\vec{V}\cdot\vec{v})}{r^2}$$

$$+ \frac{2M(\vec{V}\cdot\vec{r})v^2}{r^2}$$

$$+ \frac{4M^2(\vec{V}\cdot\vec{r})}{r^3}$$

$$+ \frac{11}{6}V^6$$

$$- \frac{5}{8}V^4(\vec{V}\cdot\vec{v})$$

$$+ \frac{1}{2}V^2(\vec{V}\cdot\vec{v})^2$$

$$- (\vec{V}\cdot\vec{v})^3$$

$$- \frac{1}{8}V^4 v^2$$

$$- \frac{1}{2}V^2(\vec{V}\cdot\vec{v})v^2$$

$$+ (\vec{V}\cdot\vec{v})^2 v^2$$

$$+ \frac{2M(\vec{V}\cdot\vec{r})^4}{r^5}$$

$$+ \frac{2M V^2(\vec{V}\cdot\vec{r})^2}{r^3}$$

$$- \frac{6M(\vec{V}\cdot\vec{r})^2(\vec{V}\cdot\vec{v})}{r^3}$$

$$+ \frac{2M(\vec{V}\cdot\vec{r})^2 v^2}{r^3}$$

$$+ \frac{6M^2(\vec{V}\cdot\vec{r})^2}{r^4}$$

$$+ \frac{2M V^2(\vec{V}\cdot\vec{v})}{r}$$

$$- \frac{2M(\vec{V}\cdot\vec{v})^2}{r}$$

$$- \frac{6M(\vec{V}\cdot\vec{r})^2(\vec{V}\cdot\vec{v})}{r^3}$$

$$+ \frac{2M(\vec{V}\cdot\vec{r})^2 v^2}{r^3}$$

$$+ \frac{6M^2(\vec{V}\cdot\vec{r})^2}{r^4}$$

$$+ \frac{2M V^2(\vec{V}\cdot\vec{v})}{r}$$

$$- \frac{2M(\vec{V}\cdot\vec{v})^2}{r}$$

$$+ \frac{2M(\vec{V}\cdot\vec{v}) v^2}{r}$$

$$- \frac{M V^2 v^2}{r}$$

$$+ \frac{M^2 V^2}{r^2}$$

$$- \frac{6M^2(\vec{V}\cdot\vec{v})}{r^2}$$

$$+ \frac{2M^2 v^2}{r^2}$$

$$+ \frac{4}{3}\frac{M^3}{r^3}$$